Awareness Equilibrium

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Abstract

We provide a model of awareness in normal-form games with incomplete information. Players and actions are common knowledge but types and probability distribution over these are not. Each player is aware of a subset of types and a joint distribution over these, and holds conjectures about other players' awareness and conjectures. In an awareness equilibrium, strategies are best replies, and the ensuing outcome is consistent with players' conjectures. We compare awareness equilibrium outcomes to the Bayesian equilibrium outcomes under a common prior. In a wide class of well-studied environments (e.g., auctions), the outcomes are the same under both scenarios. In our last example we construct a game where an awareness equilibrium outcome cannot be justified in a Bayesian equilibrium under any prior.

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1 Introduction

Starting with Aumann [1976], "Agree to Disagree", various questions regarding the impact of common knowledge have been permeating economic modeling. Recently, agents' unawareness regarding some aspects of the model has received much attention.[1] The general idea of awareness is that an agent who is unaware of a fact cannot reason about it at all. Modeling an agent with such problems is a departure from the assumption of common knowledge, and our goal is to do this in a simple and tractable way, congruent with the standard choice theory. Ultimately, the question here is whether the equilibrium predictions for such agents change relative to the standard model where information structure is common knowledge.

We consider normal-form games with incomplete information. The set of players and the action set of each player are common knowledge, but payoff-relevant types, and the joint distribution over types, are not. Awareness of a player is described simply by a subset of types (the types he is *aware* of),[2] and a conjecture about the joint distribution over these types. Such player need not assume that the other players think the same – the player holds a conjecture about awareness of other players, which is his *first-order* conjecture. Similarly, the player holds second-order conjectures (those about first-order conjectures of others), and so on *ad infinitum*. We call this construction an awareness architecture, and we impose the restriction that a player who is not aware of something cannot reason about it.[3] A primitive of our model is a set of possible awareness architectures that each player may hold. If the modeler has no idea about players' awareness then all architectures might be possible. Alternatively, the set of possible architectures embodies some information that the modeler has. For instance, a contract between two agents might reveal to

[1] To name a few, Geanakoplos [1989], Dekel et al. [1998], Halpern [2001], Feinberg [2005] and [2007], Heifetz et al. [2006] and [2007], Li [2006], Modica and Rustichini [1999], Sadzik [2006]

[2] Whenever a player is aware of some types, he is aware of the payoff consequences of those types.

[3] For example, if an agent is not aware of a certain type, he cannot conjecture that others are. Regarding the conjectures about joint distribution, this restriction is simply about its domain.

the modeler that the contingencies contracted upon are common knowledge.

Given that in our approach the modeler has potentially little information regarding possible awareness architectures, the predictions of the model could be weak. The model is then tightened by a notion of equilibrium: given awareness architectures, players' strategies are best replies, and awareness architectures are consistent with the induced outcome.[4] Of course, all of this depends on some *true* joint distribution from which types are drawn - in this sense, we assume some objective uncertainty. An *awareness equilibrium* is therefore a metaphor for a stationary situation where, if outcomes were observed, the players would not be surprised given their conjectures and the assumption that all players optimize. An outcome from the point of view of a player is the distribution over the actions of the opponent, for each realization of own action, type, and payoff, where payoff could depend on the opponent's types.

Our definition of awareness equilibrium is analogous to many other standard notions of equilibrium which require that agents' conjectures are correct.[5] For example, in the definition of a sequential equilibrium, different nodes in the game tree have to be reached exactly with the probabilities with which the players thought they would be. Alternatively, one can think of awareness equilibrium as a steady state of some learning process. Fudenberg and Levine [1993] provide such a rationale for Nash equilibrium and the *self-confirming* equilibrium for extensive-form games, and this is precisely what we have in mind. Still, in order to play the equilibrium, the players here would also have to learn the appropriate awareness architecture, so that the Fudenberg and Levine results do not apply.[6]

Relative to most of the existing literature on unawareness, our approach entails two main differences. On the one hand, we do not restrict our study to the situa-

[4]In this sense, the architectures *support* the strategies chosen by the players. As an analogy, in a rational expectations equilibrium, the price function supports an equilibrium.

[5]Nash equilibrium [1950], Conjectural equilibrium studied by Battigalli and Guatoli [1988], Rubinstein and Wolinsky [1994], self-confirming equilibrium by Fudenberg and Levine [1993], etc.

[6]We do not model such steady-state learning process formally in this paper and it is beyond the scope of this paper.

tions where the modeler can pinpoint the awareness architectures. As we mentioned this may weaken the predictions of the model. On the other hand, in equilibrium, awareness architectures have to satisfy consistency requirements, closing the model in a standard way. The question is then how these equilibrium predictions compare to the predictions of the standard model where the type space and the prior are common knowledge. We consider this comparison in terms of the modeler's knowledge regarding the prior and what he can observe regarding the outcomes.

We define three notions of outcome-equivalence. The strongest form of equivalence is when an awareness-equilibrium outcome (the joint distribution over actions and payoffs) may be justified as a Bayesian equilibrium under the *true* prior. We show that this *strong outcome equivalence* obtains in a large class of interesting environments: all private-value environments, and all environments where payoff of a player always depends non-trivially on the type draw of the opponent. These environments include auctions, double auctions, and others. In some other environments, the outcome may be justified as a Bayesian equilibrium under *some* prior, which would correspond to positive analysis with no information about the prior. Finally, the weakest form of equivalence is one in which the modeler only observes the joint distribution over players' actions (and not their payoffs, which may depend on actual type realizations), and tries to justify that as a Bayesian equilibrium given the payoff structure of the game. We carefully construct an example showing that even this weakest form of outcome equivalence need not hold in general. In that example, we indeed use the whole power of our model – in the corresponding awareness equilibrium players need to hold different conjectures at every order.

Our work thus sheds light on how crucial the assumption of common knowledge is for equilibrium prediction. In this respect we can also ask whether it is enough to consider a model where players agree on the type space but just do not hold a common prior. As we point out in the last section, this is indeed possible, and one can then simply put a zero probability (in the conjectured probability that a player makes) on types which the player is not aware of. Similarly, one could think of the awareness architecture as a specific infinite hierarchy of beliefs and indeed, this reinterpretation of our model is possible. However, the metaphor of awareness renders a simpler description, since one need not worry about ascribing actions to types with zero probability, and in equilibrium conjectures on zero-probability events are irrelevant anyway.

While we have highlighted the difference between this work and other literature on awareness, we especially comment on closely related work by Feinberg (Feinberg [2005] and Feinberg [2007]). Feinberg [2005] was the first to model awareness with a construction like our awareness architecture. In his follow-up work, he independently proposes a similar model of awareness in games with incomplete information. It is in our view reassuring that our formulations point to a similar framework. On the other hand, the motivations for our work are different, which is apparent both in the definition of our notion of awareness equilibrium, as well as in different directions, and consequently results.

2 Model

Let $N = \{1, 2\}$ be the set of agents, let $A = A_1 \times A_2$ be the set of action profiles, where A_n is finite for each $n \in \{1, 2\}$, a_n is a typical element of A_n, and a profile $a = (a_1, a_2) \in A$. Let $T = T_1 \times T_2$ be the set of players' types, where T_n is finite, $t_n \in T_n$, and $t = (t_1, t_2) \in T$. There is a $P : T \to [0, 1]$, which is the true joint probability distribution over players' types. The set of outcomes is $T \times \Delta(A_1) \times \Delta(A_2)$, the payoffs corresponding to deterministic outcomes are given by $u : T \times A \to R^2$, and the players are expected-utility maximizers. We denote by E_μ the expectation operator with respect to measure μ. We refer to (N, A, T, P, u) as a normal-form Bayesian game.

For each player, his awareness is represented by a subset of the type space and a probability distribution over it. Denote by $T^{(n)} = T_1^{(n)} \times T_2^{(n)} \subset T$ the type-awareness of player n, so that $T_1^{(n)}$ are types of player 1 that player n is aware of, $n \in \{1, 2\}$. A player is aware of $u(a,t) = (u_1(a,t), u_2(a,t))$ if and only if he is aware of t. Along with $T^{(n)}$, player n also conjectures a distribution $P^{(n)}$ over $T^{(n)}$. Zeroth order conjecture of agent n (or his awareness) is thus $(T^{(n)}, P^{(n)})$.

Awareness architectures. Players make conjectures about others' awareness and others' conjectures and so on *ad infinitum*. These conjectures have to satisfy certain requirements, which provide a natural description of awareness. We call this construction an awareness architecture. We assume that players are sophisticated in the sense that they understand that the axioms which describe their own reasoning also describe the reasoning of other agents, and that this is common knowledge. To stress this, we always explicitly specify how each axiom operates at higher orders of conjectures.

Let $(T^{(n,m)}, P^{(n,m)}) = (T_1^{(n,m)} \times T_2^{(n,m)}, P^{(n,m)})$ denote the first-order conjecture of player n about m's awareness $(T^{(m)}, P^{(m)})$, where $P^{(n,m)}$ is a joint distribution over $T^{(n,m)}$. Similarly, we define higher-order conjectures. Denote the awareness architecture of agent n by $c_n = (T^{(n)}, P^{(n)}, T^{(n,m)}, P^{(n,m)}, T^{(n,m,n)}, P^{(n,m,n)}, ...)$, $n, m \in N$. We require that c_n satisfies two basic properties, that agents make correct conjectures about own awareness, and agents can only make conjectures about types they are aware of.[7] For a finite sequence k of length x, let (k, n) be a sequence of length $x+1$, such that $(k,n)_i = k_i$ for each $i \leq x$ and $(k,n)_{x+1} = n$.

Assumption 1. $\emptyset \neq T_l^{(k,m)} \subset T_l^k$, and if $k = (n, ..., m)$, then $T^{(k,m)} = T^k$, $\forall l \in N$, for all $k \in N^x, \forall x < \infty$, $n, m \in N$.

Define by $\bar{C}_n \subset \left(\{0,1\}^T \times \{0,1\}^{\Delta(T)}\right) \times \left(\{0,1\}^T \times \{0,1\}^{\Delta(T)}\right) \times ...$ the set of all awareness architectures of player n which satisfy A1. The set of all possible awareness architectures for player n is $C_n \subset \bar{C}_n$. [8] It is often more convenient to

[7]Both of these correspond to axioms in Feinberg [2005], namely, the Weak Awareness Axiom.

[8]As mentioned in the introduction, $C_n = \bar{C}_n$ describes a situation where the modeler has no

write $C_n = C_{n,T} \times C_{n,P}$, where $C_{n,T} \subset \bar{C}_{n,T} \subset \{0,1\}^T \times \{0,1\}^T \times ...$, and $C_{n,P} \subset \bar{C}_{n,P} \subset \{0,1\}^{\Delta(T)} \times \{0,1\}^{\Delta(T)} \times ....$ The space of possible awareness architectures is $C = C_1 \times C_2$. A game with type awareness is $\mathcal{U} = (N, A, u, C)$. This collection describes the primitives of the model.

Strategy architectures. A strategy for player n is a mapping $s_n^{(n)} : T_n^{(n)} \to \Delta(A_n)$, where $\Delta(A_n)$ is the set of probability distributions over A_n. Player n conjectures a strategy of player m, which is a mapping $s_m^{(n)} : T_m^n \to \Delta(A_m)$. Let $s^{(n)} = (s_n^{(n)}, s_m^{(n)})$. At the next order, n conjectures that m conjectures a strategy of n, $s_n^{(n,m)} : T_n^{(n,m)} \to \Delta(A_n)$, and own strategy $s_m^{(n,m)} : T_m^{(n,m)} \to \Delta(A_m)$. Let $s^{(n,m)} = (s_n^{(n,m)}, s_m^{(n,m)})$, and so on at higher orders of conjectures. We define $s_n = (s^{(n)}, s^{(n,m)}, ...)$, as the *strategy architecture* of player n, and we call $s = (s_n, s_m)$ the strategy-architecture profile (from now on, a profile). The payoff-relevant components of a profile s are $(s_n^{(n)}, s_m^{(m)})$. Analogous to A1, a strategy of a player is defined only over types he is aware of. The following requirement describes this restriction on conjectures, again with sophisticated players.

Assumption 2. $s^{(k,n)} : T^{(k,n)} \to \Delta(A_n)$ and $s_m^{(k,n,m)} = s_m^{(k,n)}$.

Denote by $P(t_n)$ the marginal probability of t_n under the joint distribution P, i.e., $P(t_n) = \sum_{t_m \in T_m} P(t_n, t_m)$. Let V_n be the set of all possible utility payoffs that player n can obtain, given types and actions of both players, i.e., $v \in V_n$ iff there exist $a \in A$, and $t \in T$ s.t. $v = u_n(a, t)$. Denote by $\Pr(a_m \mid s_m^{(n)}, P^{(n)}, t_n)$ the probability of action $a_m \in A_m$, conditional on t_n, and given the conjectures $s_m^{(n)}, P^{(n)}$ (similarly for other orders of conjectures). Finally, denote by $\Pr(a_m, v \mid s_m^{(n)}, P^{(n)}, t_n)$ the joint probability of action $a_m \in A_m$ and payoff $v \in V_n$, given $t_n, s_m^{(n)}, P^{(n)}$, and analogously for higher orders of conjectures.

additional information about players' awareness. $C_n \neq \bar{C}_n$ says that some awareness architectures are ruled out, e.g., if the modeler knew there was a public announcement about the existence of some of the types.

Definition 1. An *awareness equilibrium* (AE) is a profile s and awareness architectures $(c_1, c_2) \in C_1 \times C_2$ such that

AE1 If $P(t_n) > 0$ then $t_n \in T_n^{(n)}$ and $P^{(n)}(t_n) = P(t_n), \forall t_n \in T_n$.
If $P^{(k,n)}(t_m) > 0$ then $t_m \in T_m^{(k,n,m)}$ and $P^{(k,n,m)}(t_m) = P^{(k,n)}(t_m), \forall t_m \in T_m^{(k,n)}$, $\forall m, n \in N, \forall k \in N^x, \forall x < \infty$.

AE2 $\Pr(a_m, v \mid s_m^{(k,n)}, P^{(k,n)}, t_n) = \Pr(a_m, v \mid s_m^{(m)}, P^{(k)}, t_n)$, $\forall a_m \in A_m$, $\forall m, n \in N, m \neq n, \forall v \in V_n, \forall t_n \in T_n^{(k,n)} \forall k \in N^x, \forall x \geq 0$.

AE3 $s_n^{(k,n)} = \arg\max_{\hat{s}_n^{(k,n)}: T_n^{(k,n)} \to \Delta(A_n)} E_{s_m^{(k,n)}} E_{P^{(k,n)}|t_n} \left[u(\hat{s}_n^{(k,n)}, s_m^{(k,n)}, t_n, t_m) \right], \forall t_n \in T_n^{(k,n)}$, $k \in N^x, \forall x < \infty$, $n, m \in N$.

AE1 and AE2 require that players' awareness and strategy architectures are consistent with their observations, which in particular, allows the players to make empirically correct inference. For example, in an awareness equilibrium, the players should never be surprised by a correlation of observables, which they previously thought was not there.

AE1 requires that awareness architectures are internally consistent, and that for each player his conjecture about the frequencies of his own types is correct. The interpretation is that a player should not be surprised in any event which implies that the conjectured marginal distribution over his own types is correct. At higher orders the statement of AE1 is a consequence of sophistication. That is, at each order, a player's conjecture about the distribution over types is consistent with the marginal distribution over his types under the distributional conjecture at the previous order.

AE2 requires that conditional on his own type, a player's strategy architecture, along with his awareness architecture, is consistent with the joint distribution over the actions of the other player and his own payoffs, at every order, which is precisely what each player is able to empirically verify. To illustrate, suppose that at zeroth order, player 1 makes a conjecture $P^{(1)}, T^{(1)}$, and $s^{(1)}$. This induces a (conjectured) probability distribution over 2's actions and 1's payoffs, conditional on his type. AE2 requires that this be consistent with the distribution over 2's actions and 1's payoffs, conditional on 1's type, induced by 2's true strategy, and the true probability distribution over types. Similarly, at the first order, player 1 must conjecture that player 2's observation (induced by $P^{(1,2)}$, $s^{(1,2)}$, $T^{(1,2)}$), equals to what player 1 thought player 2's observation should be (as induced by $P^{(1)}$, $s^{(1)}$, $T^{(1)}$).

AE3 requires that players are best responding to the perceived randomization over the opponent's actions, and they conjecture at every order that players are best replying. That is, it is common knowledge that players optimize.

In summary, in an AE, players may hold wrong conjectures about types, probability distribution over types, and the strategies that each is playing, as long as these are empirically consistent, and consistent with optimizing behavior.Our departure from the standard model is to relax the assumption of common knowledge of the type space and priors, which is formalized by c_n. We do not change the assumption of players' rationality, which is embodied in equilibrium requirements AE1-AE3. That is, in our view rationality is the ability to optimize in the perceived choice set, and to do so it is also necessary to correctly assess the joint distribution over this perceived choice set which comes from some objective uncertainty potentially unknown to players. For otherwise these players would be without a doubt wrong in their optimization procedure and only one of the following two possibilities could occur. One, they would have to figure out that they are wrong and deviate, or two, they would be drawing logically incorrect conclusion in terms of their inference, and would thus not be rational.

How far is this model from a standard Bayesian game? If $C_n = \{T \times P \times T \times P...\}, \forall n \in N$, then (N, A, u, C) is equivalent to the Bayesian game (N, A, T, P, u), and AE is equivalent to Bayesian equilibrium. Indeed, A1 and A2 hold, AE1 is vacuously satisfied, AE2 is the standard requirement that in equilibrium beliefs are correct, and AE3 requires that agents play best replies.

Example 1. **Coordination-anticoordination game.** Let $N = \{1, 2\}, A_1 = \{up, down\}, A_2 = \{L, R\}, T_1 = \{+\}, T_2 = \{+, -\}, P(+, +) = \pi, P(+, -) = 1-\pi, \pi \geq \frac{2}{3}$, and the payoffs be given by:

$(t_1 = t_2 = +)$	L	R
up	$1, 1$	$0, 0$
down	$0, 0$	$2, 2$

$(t_1 = -t_2)$	L	R
up	$1, 0$	$0, 2$
down	$0, 1$	$2, 0$

In the Bayesian game, player 1's mixed strategy is $(\sigma, 1 - \sigma)$, where $\sigma \in [0, 1]$ is the probability of playing *up*. Player 2's mixed strategy is $(\sigma_+, 1 - \sigma_+, \sigma_-, 1 - \sigma_-)$, where σ_t is the probability of playing L if $t_2 = t$. Bayesian equilibria are $(\sigma = 0; \sigma_+ = 0; \sigma_- = 1)$, $(\sigma = 1; \sigma_+ = 1; \sigma_- = 0)$, $\left(\sigma = \frac{2}{3}; \sigma_+ = \frac{2}{3\pi}; \sigma_- = 0\right)$.

If C only includes architectures where $T_2^{(1)} = \{-\}$ then no AE exist. Indeed, under such conjecture and 1's randomization of *up* with probability $\frac{2}{3}$, in 1's view player 2 does not play a best reply by randomizing, so that AE3 would be violated.

If C includes architecture where $T_2^{(1)} = \{+\}$, and $P^{(1)}(t_2 = +) = 1$, then the only AE outcome is the outcome of the Bayesian equilibrium $\left(\sigma = \frac{2}{3}; \sigma_+ = \frac{2}{3\pi}; \sigma_- = 0\right)$. Indeed, if 2 plays $\left(\sigma_+ = \frac{2}{3\pi}; \sigma_- = 0\right)$, then 1 observes L with probability $\pi\frac{2}{3\pi} = \frac{2}{3}$, and R with probability $\frac{1}{3}$. Since $T_2^{(1)} = \{+\}$, 1 is indifferent between *up* and *down*, and playing $\sigma = \frac{2}{3}$ is a best reply that in 1's view makes 2 indifferent between his actions, hence $s_2^{(1)} = (\frac{2}{3}, \frac{1}{3})$ satisfies AE3. We can similarly construct higher-order conjectures for player 1, and there is some freedom in constructing conjectures for player 2.[9]

In light of Example 1 we comment that whenever C contains the common-prior architecture, AE exist. In particular, AE exist if $C = \bar{C}$ and such model exhausts the set of AE.[10]

[9] Player 2 could either have correct conjectures about player 1's awareness architecture, or he could conjecture that player 1 is fully aware of 2's types.

[10] In the appendix we provide a tight characterization of C for the existence of AE for the class of private-value games.

3 AE and Bayesian equilibrium

The question we explore is how the outcomes of AE compare to the Bayesian equilibrium outcomes. This comparison may be either in terms of behavior or the outcome observed by an outsider. We define three notions of equivalence between AE and Bayesian equilibrium.

DEFINITION **2.** Let (s, c) be a AE of (N, A, u, C) and let s^* be a Bayesian equilibrium of $(N, A, T, \tilde{P}, u)$.

1. We say that (s, c) and s^* are *observationally equivalent* if $\Pr(a \mid s_1^{(1)}, s_2^{(2)}, P) = \Pr(a \mid s^*, \tilde{P}), \forall a \in A$, i.e., if the joint distribution over actions is the same in both equilibria.

2. We say that (s, c) and s^* are *weakly outcome equivalent* if $\Pr(a, v \mid s_1^{(1)}, s_2^{(2)}, P) = \Pr(a, v \mid s^*, \tilde{P}), \forall a \in A, \forall v \in V_1 \times V_2$, i.e., if the joint distribution over actions and payoffs is the same in both equilibria.

3. We say that (s, c) and s^* are *strongly outcome equivalent* if $\Pr(a, v \mid s_1^{(1)}, s_2^{(2)}, P) = \Pr(a, v \mid s^*, \tilde{P}), \forall a \in A, \forall v \in V_1 \times V_2$, and $\tilde{P} = P$.

We remark that *strong outcome equivalence* $\Rightarrow$ *weak outcome equivalence* $\Rightarrow$ *observational equivalence*, but the reverse implications are not true.

Observational equivalence describes a situation where an outside observer knows the payoff structure and asks whether it is possible to justify observed behavior as Bayesian equilibrium behavior under some prior. Under weak outcome equivalence, the observer also knows the equilibrium payoffs that were attained by players. Under strong outcome equivalence, the observer knows the true distribution over the states of the world.

We first show that there is a wide class of environments in which strong outcome equivalence obtains. We say that a game satisfies *private values* if $u_n(a, t_n, t_m) = u_n(a, t_n, t'_m), \forall t_m, t'_m \in T_m, \forall t_n \in T_n, \forall a \in A$. We say that a game satisfies *generic interdependence* if $u_n(a, t_n, t_m) \neq u_n(a, t_n, t'_m), \forall t_m, t'_m \in T_m, \forall t_n \in T_n, \forall a \in A$.

Proposition **1.** Let (N, A, u, C) be a type-awareness game. If it satisfies either private values or generic interdependence then all its AE are strongly outcome equivalent to Bayesian equilibria of (N, A, T, P, u).

Proof. Take private values, and let (s, c) be a AE. By AE3, $s_1^{(1)}$ is a best reply to $s_2^{(1)}$ under the distribution $P^{(1)}$. AE2 implies that

$$\Pr(a_2 \mid t_1, s_2^{(1)}, P^{(1)}) = \Pr(a_2 \mid t_1, s_2^{(2)}, P), \forall t_1 \in T_1, \forall a_2 \in A_2.$$

By private values, $u_1(a, t_1, t_2) = u_1(a, t_1), \forall (t_1, t_2) \in T, \forall a \in A$. Using these two observations, we have that

$$E_{s_2^{(1)}, P^{(1)}} u_1(a, t_1, t_2) = E_{s_2^{(2)}, P} u_1(a, t_1), \forall a_1 \in A_1.$$

Thus, $(s_1^{(1)}, s_2^{(2)})$ is a Bayesian equilibrium of (N, A, T, P, u), which implies strong outcome equivalence.

Now generic interdependence. Take a $v_1 \in V_1$ (recall that V_1 is the set of utility payoffs for player 1), then $\exists a \in A, (t_1, t_2) \in T$, s.t. $v_1 = u_1(a, t_1, t_2)$ and by generic interdependence $u_1(a, t_1, t'_2) \neq v_1, \forall t'_2 \neq t_2$. Using this, we have that

$$\Pr(v_1, a_2 \mid t_1, s_2^{(1)}, P^{(1)}) = \Pr(t_2, a_2 \mid t_1, s_2^{(1)}, P^{(1)}), \tag{1}$$

so that

$$\Pr(t_2 \mid t_1, P^{(1)}) = \sum_{a_2 \in A_2} \Pr(t_2, a_2 \mid t_1, s_2^{(1)}, P^{(1)}).$$

Similarly,

$$\Pr(t_2 \mid t_1, P) = \sum_{a_2 \in A_2} \Pr(t_2, a_2 \mid t_1, s_2^{(2)}, P).$$

By AE2 and (1),

$$\Pr(t_2, a_2 \mid t_1, s_2^{(1)}, P^{(1)}) = \Pr(t_2, a_2 \mid t_1, s_2^{(2)}, P), \forall t_2 \in T_2, \forall a_2 \in A_2,$$

which implies that $\Pr(t_2 \mid t_1, P^{(1)}) = \Pr(t_2 \mid t_1, P)$, i.e., player 1 must correctly conjecture the type distribution, and similarly at higher orders. Thus, $(s_1^{(1)}, s_2^{(2)})$ is a Bayesian equilibrium. □

The intuition here is that under private values, a player only needs to best-reply to the distribution over opponent's actions, so that his conjectures over opponent's types and marginal probability over those types are irrelevant. If the environment is not one of private values, then a player needs to make a conjecture regarding the distribution over opponent's types in order to be able to compute a best reply. Under generic interdependence, opponent's types nontrivially affect his own payoffs, which he observes, so that such conjecture must be correct. Consequently, in an AE the true prior needs to be common knowledge (modulo types with zero probabilities, of which the players do not need to be aware). As a remark, it is then clear that an AE exists if and only if C includes such architecture. This is in contrast with the case of private values, for which an AE may exist even if C doesn't include the common-prior architecture – see Example 1 and propositions 3 and 4 in Appendix.

Proposition 1 shows that in these environments, relaxing players' knowledge requirements regarding the prior and the type space does not alter equilibrium predictions. Hence, the modeler does not need to worry about awareness of the players in order to make predictions.

On the one hand, the conditions on the primitives describing generic interdependence and private values are very easy to verify. On the other hand, these environments are widely studied in Economics. To name just a few examples: auctions and double auctions, many standard examples of oligopolistic competition, but also non-market settings such as provision of public goods, and so on.[11] An immediate corollary of Proposition 1 is that the Milgrom and Stokey [1982] "No-trade" theorem always obtains in AE.

In Example 2 we show that strong outcome equivalence doesn't always obtain, i.e., an outcome of a AE need not be sustainable in a Bayesian equilibrium under the true prior. However, in this example, the outcome of that AE is sustainable in a Bayesian equilibrium under *some other* prior, so that the weak outcome equivalence still obtains.

EXAMPLE **2.** [12]

$N = \{1,2\}, A_1 = \{up, down\}, A_2 = \{L\}, T_1 = \{t_1\}, T_2 = \{t_2, t_2'\}$,
$P(t_1,t_2) = \frac{3}{4}, P(t_1,t_2') = \frac{1}{4}$, and the payoffs are given by the following two payoff matrices:

(t_1,t_2)	L
up	$1,0$
down	$0,0$

(t_1,t_2')	L
up	$-2,0$
down	$0,0$

Let $T^{(k)} = T, \forall k$, $P^{(1)}(t_1,t_2) = P^{(1)}(t_1,t_2') = \frac{1}{2}$, $P^{(2)} = P$, and let $P^{(k)} = P^{(1)}, \forall k \in N^x, x \geq 2$. Now let $s^{(k)} = (down, L)$. Then (c,s) thus defined constitutes a AE, which is easy to check. It is clear that $(down, L)$ is not a Bayesian equilibrium under P. However, $(down, L)$ is a Bayesian equilibrium under $\tilde{P} = P^{(1)}$.

In the situation described by the above example, under specific conjectures, optimizing actions of players may make these conjectures self confirming, and because of

[11]For instance, the auction environments in Milgrom and Weber [1982] fall under Proposition 1, as well as oligopolistic Bertrand competition with incomplete information on the marginal cost of production as in Spulber [1995], and double auction models such as in Myerson and Satterthwaite [1983].

[12]The game is from Example 5 (Affirmative Action), Jackson and Kalai [1997].

the payoff structure at least one player cannot figure out that these conjectures differ from the true prior. The following proposition slightly generalizes this intuition.

PROPOSITION **2.** Suppose that for $m, n \in N, m \neq n$, there exist $t_m \in T_m,\ t_n, t'_n \in T_n, a \in A$ s.t. the following conditions hold:

1. $a_m \in \arg\max_{a'_m \in A_m} u_m(a_n, a'_m, t_n, t_m)$ and $a_m \notin \arg\max_{a'_m \in A_m} u_m(a_n, a'_m, t'_n, t_m)$,
2. $a_n \in \arg\max_{a'_n \in A_n} u_n(a'_n, a_m, t_n, t_m) \cap \arg\max_{a'_n \in A_n} u_n(a'_n, a_m, t'_n, t_m)$,
3. $u_m(a, t_n, t_m) = u_m(a, t'_n, t_m)$.

Then, under unrestricted C, there exists a P, and a AE of (N, A, u, C), which is not strongly outcome equivalent to any Bayesian equilibrium of (N, A, T, P, u).

Condition 1 says that there should exist two types of player n, such that for each of these player m has a different best reply. Condition 2 and 3 state that in order for m to not be able to discern the relative likelihoods of these two types, n must play the same action for both, and m must obtain the same payoffs. It easy to verify that in Example 2, these conditions are satisfied by taking $n = 2$, $m = 1$.

Proof. Assume wlog that $n = 1, m = 2$, and let P be such that $P(t_1, t_2) + P(t'_1, t_2) = 1$ and $P(t_1, t_2)u_2(a_1, a_2, t_1, t_2) + P(t'_1, t_2)u_2(a_1, a_2, t'_1, t_2) < P(t_1, t_2)u_2(a_1, a'_2, t_1, t_2) + P(t'_1, t_2)u_2(a_1, a'_2, t'_1, t_2)$ (condition 1 implies that A_2 has at least two elements, and that it is possible to find such an a'_2).

Let $\tilde{P}$ be such that $\tilde{P}(t_1, t_2) + \tilde{P}(t'_1, t_2) = 1$ and

$$\begin{aligned}
&\tilde{P}(t_1, t_2)u_2(a_1, a_2, t_1, t_2) + \tilde{P}(t'_1, t_2)u_2(a_1, a_2, t'_1, t_2) \\
&\geq \tilde{P}(t_1, t_2)u_2(a_1, a'_2, t_1, t_2) + \tilde{P}(t'_1, t_2)u_2(a_1, a'_2, t'_1, t_2), \forall a'_2 \in A_2, a'_2 \neq a_2.
\end{aligned}$$

Now set $T^{(k)} = \{(t_1, t_2), (t'_1, t_2)\},\ P^{(1)} = P, P^{(k)} = \tilde{P}, \forall k \neq (1),\ s_1^{(k)} = a_1, s_2^{(k)} = a_1,\ \forall k$. It is immediate to verify that (s, c) is an AE, and by construction, $(s_1^{(1)}, s_2^{(2)})$

is not a Bayesian equilibrium under P, implying that strong outcome equivalence doesn't hold. $\square$

4 Failure of observational equivalence

One might be tempted to believe that weak outcome equivalence always obtains. We provide an example where we show that even observational equivalence doesn't always hold, i.e., there exist games with AE, in which the observed distribution over players' actions is not supportable as a Bayesian equilibrium under any prior. Thus, an outside observer who observes a joint distribution over rational players' actions will reject the hypothesis of common knowledge of the type space and prior.

The example is constructed on two principles. First, the three conditions of Proposition 2 are satisfied for both players in each role. This allows to sustain an outcome in an AE, in which each player is unable to discern relative likelihoods of two draws of types, call them (t_1, t_2), (t_1, t_2') for player 1, and $(t_1, t_2), (t_1', t_2)$ for player 2. Second principle is to construct payoffs in such a way that this AE outcome cannot be sustained as a Bayesian equilibrium under any prior. Payoffs have to be such that player 1 must conjecture that (t_1, t_2) is sufficiently unlikely relative to (t_1, t_2'), while 2 must conjecture that (t_1, t_2) is sufficiently likely, relative to (t_1', t_2). We can then construct conjectures on type distribution which at every order rationalize own and opponent's behavior and also satisfy the consistency requirements of AE1 and AE2. It is important to note that contrary to the awareness architectures in previous examples, in this example the conjectures are different at every order. Still, at no order is a player ever surprised with the result.

Example **3.** Let each player have two types, $T_n = \{t_n, t_n'\}$, and two actions, and let the payoff structure for each draw of types be specified as follows.

(t_1, t_2)	L	R
u	$0, 6$	$0, 0$
d	$0, 0$	$-10, 0$

(t_1, t_2')	L	R
u	$0, 0$	$0, 1$
d	$0, 0$	$4, 0$

(t_1', t_2)	L	R
u	$0, -1$	$1, 0$
d	$0, 0$	$3, 0$

(t_1', t_2')	L	R
u	$0, 30$	$2, 0$
d	$0, 0$	$-50, 0$

The true prior distribution P is

P	t_2	t_2'
t_1	$\frac{3}{10}$	$\frac{3}{10}$
t_1'	$\frac{3}{10}$	$\frac{1}{10}$

We first construct an AE in which each type of player 1 plays u, and each type of player 2 plays R. Denote this strategy profile by $s^* = (s_1^{(1)}, s_2^{(2)})$. We then show that there does not exist a common prior $\tilde{P}$ under which this strategy profile could be a Bayesian equilibrium.

Construction of AE. We construct $P^{(1)}$ and $P^{(2)}$ to sustain s^* in an AE. First, to construct the strategy architectures note that by AE2, it has to be that $s^{(k)} = s^*$, for all $k \in N^x, x > 0$.

Notice that when player 2 plays R, and player 1 is of type t_1', player 1 must by AE2 correctly conjecture the conditional distribution on player 2's types, i.e., $P^{(1)} \mid_{t_1'} (x) = P \mid_{t_1'} (x)$, $x \in \{t_2, t_2'\}$. This is so because player 1 obtains different payoffs for different types of player 2. Similarly, player 2 must correctly conjecture the conditional distribution on player 1's types when he is type t_2'.

By AE3, in order for t_1 to play u when 2 plays R, $P^{(1)}$ must satisfy the following.

$$-10P^{(1)}(t_1, t_2) + 4P^{(1)}(t_1, t_2') \leq 0, \tag{2}$$

Similarly, for t_2 to play R when 1 plays u, $P^{(2)}$ must satisfy

$$6P^{(2)}(t_1,t_2) - 1P^{(2)}(t'_1,t_2) \leq 0. \tag{3}$$

Also, by AE1, $P^{(1)}(x,t_2)+P^{(1)}(x,t'_2) = P(x,t_2)+P(x,t'_2)$, $x \in \{t_1,t'_1\}$ and $P^{(2)}(t_1,x)+P^{(2)}(t'_1,x) = P(t_1,x)+P(t'_1,x)$, $x \in \{t_2,t'_2\}$. All these conditions are satisfied by the following $P^{(1)}$, $P^{(2)}$.

$P^{(1)}$	t_2	t'_2
t_1	$\frac{2}{10}$	$\frac{4}{10}$
t'_1	$\frac{3}{10}$	$\frac{1}{10}$

$P^{(2)}$	t_2	t'_2
t_1	0	$\frac{3}{10}$
t'_1	$\frac{6}{10}$	$\frac{1}{10}$

Second, define the higher order conjectures on P inductively as follows. For $k \in N^x, x \geq 0$, given $P^{(k,1)}$, define $P^{(k,1,2)}$ such that $P^{(k,1,2)}(t_1,t_2) + P^{(k,1,2)}(t'_1,t_2) = P^{(k,1)}(t_1,t_2) + P^{(k,1)}(t'_1,t_2)$, and such that (3) is satisfied, and set $P^{(k,1,2)}(t_1,t'_2) = P^{(k,1)}(t_1,t'_2)$, $P^{(k,1,2)}(t'_1,t'_2) = P^{(k,1)}(t'_1,t'_2)$. In this way, $s_2^{(k,1)}$ and $P^{(k,1,2)}$ satisfy AE3, and by construction AE1 is satisfied by $P^{(k,1,2)}$. Analogously, given $P^{(k,2)}$, construct $P^{(k,2,1)}$, such that condition (3) holds (implying that AE3 holds for $s_1^{(k,2)}$ and $P^{(k,2,1)}$) and AE1 holds. Finally, setting $c_n = (T, P^{(n)}, T, P^{(n,m)}, ...)$, $s^{(k)} = s^*$, $\forall k \in N^x, x > 0$, (s,c) is an AE.

Failure of observational equivalence. Now we claim that there does not exist a common prior $\tilde{P}$, such that in the game $(N, A, u, T, \tilde{P})$, s^* is a BE.

We first show that for no $\tilde{P}$ that puts mass 1 on a single type of one player s^* is a BE. We have to consider 10 different cases. The 4 cases where $\tilde{P}$ puts mass 1 on a single draw of types are trivial to check, and the other 6 cases are also simple. To illustrate the logic take for example a $\tilde{P}$ that puts mass 1 on $\{(t_1,t_2),(t'_1,t_2)\}$: if player 1 is type t'_1 and player 2 plays R, then player 1 would play d.

Next, for every $\tilde{P}$ that puts 0 probability on exactly one draw of types, it is also easy to check in the same way that s^* is not supportable in a BE. The last case is when $\tilde{P}$ puts positive probability on all draws. Denote $p_1 = \tilde{P}(t_1, t_2)$, $p_2 = \tilde{P}(t_1, t_2')$, $p_3 = \tilde{P}(t_1', t_2)$, $p_4 = \tilde{P}(t_1', t_2')$, and write the incentive constraints,

$$0 \geq 4p_2 - 10p_1, \tag{4}$$

$$0 \geq 2p_3 - 52p_4, \tag{5}$$

$$0 \geq 6p_1 - p_3, \tag{6}$$

$$0 \geq 30p_4 - p_2. \tag{7}$$

Add (5) and $4\times$ (7), add (6) and $2\times$ (7), and add $5\times$ the first resulting inequality to $6\times$ the second one, to obtain $0 \geq 460p_4$, which is a contradiction.

5 Conclusion

In the model of type awareness we allow for conjectures over types and probability distributions over these. One may wonder whether a model where the type space is common knowledge and conjectures are only over the distributions over types is rich enough. In principle, this is not the case, but as the following remark illustrates, it is the case in equilibrium.

Remark **1.** Consider two type awareness games, $(N, A, u, \bar{C})$, and (N, A, u, C), where $C_{n,T} = \{T\} \times \{T\} \times ...$, and $C_{n,P} = \{0,1\}^{\Delta(T)} \times \{0,1\}^{\Delta(T)}...$ for both n. In other words, T is common knowledge in (N, A, u, C). Then the set of AE of $(N, A, u, \bar{C})$ is equal to the set of AE of (N, A, u, C).

Proof. Whenever a player is not aware of some type, this is in equilibrium equivalent to putting probability 0 on that type. This shows that $\{AE \text{ of } (N, A, u, \bar{C})\} \subset \{AE \text{ of } (N, A, T, u, P, C)\}$. The other inclusion is trivial. $\square$

We remark that we avoided technical complications of explicitly modeling our types as rich types, in terms of their conjectures (i.e., different types have the same awareness architecture). However, players are allowed to entertain conjectures on probability distributions over types where the opponent's distribution varies conditional on own type. By Remark 1, a consequence is that modeling types as rich would not alter equilibrium predictions (and would add substantial layers of technical difficulty). We refer to Feinberg [2007] for a precise construction of a one-to-one map between Harsanyi types and awareness architectures.

In the present work, we explored a situation where agents can verify the joint distribution over actions of the opponent and own payoffs. But there are many interesting situations where players cannot verify actual actions taken by the other player but just some statistic of those. An example of such a situation is moral hazard, and more generally, models of hidden actions. On the one hand, our model is flexible enough to extend to such environments - one would have to modify the definition of AE slightly. On the other hand, the AE considered here will always be equilibria under those circumstances as well (just consider a situation where a player happens to *conjecture* the true distribution over the opponents actions). Nevertheless, this may be useful for possible applications of our model. It is also a necessary step for writing down a model where agents are unaware of other agents- in the present model, if an agent is aware of some other agent's actions he needs to be aware of that agent.

Finally, one may consider a model where agents lack awareness of actions. We present a very simple version of such a model in our working paper Čopič and Galeotti [2006]. Incorporating such awareness of actions in the present model of incomplete information is straightforward.

Appendix–Additional Results for Private-Value Games.

In this appendix we only consider private-value environments. The next proposition provides conditions for the existence of AE.

Proposition **3.** Consider a private-value environment. Let $C_{n,P} = \{0,1\}^{\Delta(T)} \times \{0,1\}^{\Delta(T)}$.... Then a AE exists if and only if there exists a $\bar{T} = \bar{T}_1 \times \bar{T}_2$ such that the following two properties hold.

1. There exists a Bayesian equilibrium s^* such that $\cup_{t_n \in T_n} supp\,(s_n^*(t_n)) = \cup_{t_n \in \bar{T}_n} supp\,(s_n^*(t_n))$.

2. There exists a $c_n \in C_n$ with the property that $(c_{n,T})_{(k,n)} = T_n$ and $(c_{n,T})_{(k,m)} = \bar{T}_m$, for all $k \in N^x, x \geq 0$, $n, m \in N$.

Proof. Suppose that both conditions hold, and let $\bar{T}$ be as specified in the proposition. Take player 1. We will show that if $\bar{T}_2 = T_2 \setminus \hat{t}$, for some $\hat{t}$, then there exists a AE. The general case then follows immediately by induction.

Take an action $\hat{a} \in supp\left(s_2^*(\hat{t})\right)$. Since $\bar{T}$ satisfies Condition 1, there exists a type $\bar{t} \in \bar{T}$, s.t. $\hat{a} \in supp\,(s_2^*(\bar{t}))$. Let $\alpha(\hat{a} \mid \hat{t})$ denote the probability that s_2^* assigns to $\hat{a}$ given type $\hat{t}$. Let $\bar{P}(\bar{t}) = P(\bar{t}) + \alpha(\hat{a} \mid \hat{t})P(\hat{t})$, and allocate all the new mass of type $\bar{t}$ to the play of action a. That is, define the new mixed strategy of $\bar{t}$ as follows. Let

$$\bar{\alpha}(\hat{a} \mid \bar{t}) = \frac{\alpha(\hat{a} \mid \bar{t})P(\bar{t}) + \alpha(\hat{a} \mid \hat{t})P(\hat{t})}{P(\bar{t}) + \alpha(\hat{a} \mid \hat{t})P(\hat{t})},$$

and for every other action $\bar{a} \in supp\,(s_2^*(\bar{t}))$, let

$$\bar{\alpha}(\bar{a} \mid \bar{t}) = \frac{\alpha(\bar{a} \mid \bar{t})P(\bar{t})}{P(\bar{t}) + \alpha(\bar{a} \mid \hat{t})P(\hat{t})}.$$

By construction, the aggregate empirical distribution over actions of player 2 is unchanged by this transformation. By taking all the actions of type $\hat{t}$, and repeating this procedure, we allocate all the mass of $\hat{t}$ to other types of player 2, without affecting the empirical distribution over actions of player 2. By private values, player 1 only cares about the distribution over 2's actions. This proves sufficiency of conditions 1 and 2.

The converse follows from Proposition 1. □

We say that a AE outcome *requires correct architectures* if there exists a unique AE (s, c), where $s_n^{(k)} = s_n^*, (c_{n,T})_{(k,m)} = T_m, (c_{n,P})_{(k)} = P$, for all $k \in N^x, x \geq 0$, $n, m \in N$, where s^* is a Bayesian equilibrium sustaining the given outcome. We already saw that for the case of strict common values every AE requires correct architectures. The next proposition provides a characterization of AE that require correct architectures in private-value environments.

Proposition 4. Let $C_{n,P} = \{0,1\}^{\Delta(T)} \times \{0,1\}^{\Delta(T)} ...$, and let s^* be a Bayesian equilibrium. Then the outcome of s^* requires correct architectures if and only if

$$supp\,(s_n^*(t_n)) \cap supp\,(s_n^*(\bar{t}_n)) = \emptyset, \forall t_n, \bar{t}_n \in T_n, t_n \neq \bar{t}_n. \tag{8}$$

Proof. If condition (8) holds, then the claim follows: unless both players have correct architectures, AE4 cannot be satisfied.

For the converse, if (8) doesn't hold, then we can construct a AE which does not require correct architectures. In particular, a player's conjecture about the distribution of types need not be correct. Suppose therefore that (8) doesn't hold, so there exists two types $\bar{t}, \hat{t} \in T_2$, s.t. $supp(s_2^*(\bar{t})) \cap supp(s^*(\hat{t})) = \{\bar{a}\}$. We can assume wlog that the intersection of the supports is a single action, since the same construction can be made if the intersection is larger. Now, let player 1 conjecture the correct type space of player 2, but 1's conjecture about the probability distribution be given by

$$\bar{P}(\bar{t}) = P(\bar{t}) + \epsilon, \bar{P}(\hat{t}) = P(\hat{t}) - \epsilon,$$

where as before, $P(t)$ gives the true marginal distribution of type t, and ϵ is some positive number. Define 1's conjecture on 2's strategy, $s_2^{(1)}(\bar{t})$, by defining mixing probabilities $\bar{\alpha}(. \mid \bar{t})$ and $\bar{\alpha}(. \mid \hat{t})$. First let

$$\frac{P(\bar{t})}{\bar{P}(\bar{t})}\alpha(a \mid \bar{t}) = \bar{\alpha}(a \mid \bar{t}), \forall a \in supp(s_2^*(\bar{t})) \setminus \{\bar{a}\},$$

$$\frac{P(\hat{t})}{\bar{P}(\hat{t})}\alpha(a \mid \hat{t}) = \bar{\alpha}(a \mid \hat{t}), \forall a \in supp(s_2^*(\hat{t})) \setminus \{\bar{a}\}.$$

Since mixing probabilities have to sum to 1 for each type, we have

$$\bar{\alpha}(\bar{a} \mid \bar{t}) = 1 - \frac{P(\bar{t})}{\bar{P}(\bar{t})}\left(\sum_{a \in supp(s_2^*(\bar{t}))\setminus\{\bar{a}\}} \alpha(a \mid \bar{t})\right), \text{ and}$$

$$\bar{\alpha}(\bar{a} \mid \hat{t}) = 1 - \frac{P(\hat{t})}{\bar{P}(\hat{t})}\left(\sum_{a \in supp(s_2^*(\hat{t}))\setminus\{\bar{a}\}} \alpha(a \mid \hat{t})\right).$$

Now we make ϵ small enough so that $\bar{\alpha}(a \mid t) \in (0, 1)$, for all a and t. ¿From the last two equalities, we can easily verify that

$$P(\bar{t})\alpha(\bar{a} \mid \bar{t}) + P(\hat{t})\alpha(\bar{a} \mid \hat{t}) = \bar{P}(\bar{t})\bar{\alpha}(\bar{a} \mid \bar{t}) + \bar{P}(\hat{t})\bar{\alpha}(\bar{a} \mid \hat{t}).$$

Thus, the observed probability of playing each action is the same under P and α and under $\bar{P}$ and $\bar{\alpha}$. Clearly, the other requirements of AE are also satisfied. $\square$

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