# Trade, redistribution and indeterminacy

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**Summary**. In the present paper a tractable two-sector growth model with technological externalities and many countries is considered. It is shown that the occurrence of indeterminacy, a typical side-product of externalities, may appear due to the enlargement of the markets for goods and factors. Various scenarios of progressive levels of integration are considered. In particular, it is found that the integration into a common market on which countries trade the produced good and the inputs may lead to indeterminacy even when the equilibrium under full autarchy is determinate. A similar result holds when integration only affects consumption and capital goods. However, such result doesn't occur if the inverse of relative risk aversion is a linear or concave function. We conclude that in many usual situations, as the one with CES preferences, indeterminacy and the associated fragility of expectations and financial instability, is not likely to be increased by market integration.

**Keywords:** Economic growth, Externalities, General Equilibrium, Indeterminacy, Trade, Market Integration, Common Market.

**JEL-classification numbers: F10, F11, F15, O41.**

# 1 Introduction

The existence of a common market for the produced goods and inputs is one of the usual features of globalization. In fact, economists have since a long time analyzed the economic implications for a country to sign a trade agreement or to join a common market. The classical view that on the long run being in a large market is beneficial for all partners has survived the many "revolutions" since Ricardo. However, the notion of being beneficial is a dynamic one, and the way the size of the market affects economic growth has also to be considered. In fact the endogeneisation of the mechanisms of growth in the economic models has multiple side effects. One of these is that competitive equilibria may be indeterminate. This opens the door to high sensitivity of the dynamics to expectations. The welfare implications may be large as this overall uncertainty is usually a highly undesirable property. In the present paper we address the issue of whether indeterminacy is or is not a by-product of market integration[1].

Similarly to Ghiglino and Olszak-Duquenne (2005) we consider a general equilibrium model of the type used by Bewley (1982) but with technological externalities. The model is made tractable by focussing on two-sector economies in which the technology is specified analytically as in Boldrin and Deneckere (1990) and Boldrin and Rustichini (1994). The major innovation is that we introduce several countries, abandon the representative agent assumption and admit non-linear utility functions. Citizen of one country may differ from those of the other country in respect to the share in the initial stock of capital, to the labor endowments as well as to preferences. The time discount factor is assumed to be the same across countries. Due to the structure of the model, when capital is mobile individual characteristics and heterogeneity does not affect the steady state itself as far as aggregate variables are considered. However, this model is sufficient to analyze the impact of a common market on the determinacy of the steady state. Note that the introduction of externalities in a model with heterogenous agents and/or countries may present some difficulties (see Kehoe, Levine and Romer (1990), Santos (1992) or Ghiglino (2002)).

The tractability of the model is based on some strong simplifying assumptions. The reform is modelled in a simple way as there are only three levels

[1] A similar issue is considered by Henry Wan in Chapter 6 of Wan (2004).

of integration. In the lowest level countries are completely isolated and the produced goods and inputs are traded only on domestic markets. After a first trade agreement, the countries trade the consumption good and capital on a common market with no transaction costs but labor is immobile. Finally, at the highest level of integration the consumption good and both inputs are traded on a common market, which transforms the many countries world into a unique large country: there is complete integration. This sequence of reforms is extreme on many grounds and intermediate situations could be instead considered. However, we think that the message would not be significantly affected by such generalizations. Another strong assumption is that all countries have the same basic technology. This means that we do not focus on the effect of technological diffusion. Finally, labor endowments are considered as exogenous parameters. The model may be used to describe a variety of situations in which labor is provided inelastically. It could in some cases be used to gain intuition on the behavior of a more complex model in which labor supply is endogenously determined. In this case differences in labor endowments are a shortcut to model the differences in the disutility that agents associate to labor.

We obtain several results. First, we show that the occurrence of indeterminacy depends on how labor endowments and shares in initial capital are distributed across countries, as well as on preferences and technology. Second, and this is our main result, we give an extensive characterization of the situations in which market integration leads to indeterminacy. Our main focus is a situation in which the equilibrium under autarchy is determinate and the reform consists of joining a common market for the consumption good and for one or both inputs. We show that market integration does not produce indeterminacy if the preferences belong to the HARA class or if the inverse of relative risk aversion is a concave function. This implies that in many usual situations, as in the CES case, the volatility associated to indeterminacy, and therefore sensitivity to expectations, is not likely to increase as a consequence of a common market even when growth is endogeneised.

To our knowledge, this paper is the first attempt to analyze the link between indeterminacy and the level of market integration. The present paper is related to Ghiglino and Olszak-Duquenne (2005) and Ghiglino and Sorger (2002). In the last paper, indeterminacy is shown to occur in a continuous time, endogenous growth model with externalities and many heterogeneous agents. However, the authors do not address the issue of the effect of a common market. The reason is that their analysis fail to qualify the effects of redistributions on the occurrence of indeterminacy. Another weakness is that preferences are bound to be log-linear. Ghiglino and Olszak-Duquenne (2005) considers a model like ours but with only one country. A version of the present model without externalities and with full integration is considered by Ghiglino and Olszak-Duquenne (2001). In that paper it is shown that with no externalities the distribution of labor endowments and capital shares matters in the stability properties of the steady state. As a corollary, the effect of a common market on stability is also investigated. In the present paper we do not address the issue of the link between stability and redistribution, and therefore of the effect of a common market on stability. Some of the results on stability obtained without externalities are expected to be true in the present framework as well. Nishimura and Yano (1993) is also related. Finally, Nishimura and Shimomura (2002) investigate how the standard result on the effects of trade are affected by indeterminacy when factors are immobile.

The paper is organized as follows: In section 2 the model is introduced while in Section 3 the equilibria are defined. Section 4 focuses on the occurrence of indeterminacy. Section 5 and Section 6 analyze the effects of market integration on indeterminacy. The appendix contains most of the proofs.

# 2 The model

In the present paper we consider a competitive two-sector intertemporal economy composed of two countries having identical technologies. There is no joint-production and firms produce according to constant returns production functions so that at the optimum, profits are zero. There are two produced goods, a consumption good and a capital good. The consumption good cannot be used as capital so it is entirely consumed. The capital good cannot be consumed. There are two inputs, capital and labor. We also suppose that there is instantaneous capital depreciation and that labor is inelastically used in production. Externalities are of the labor-augmenting type as detailed below.

The technology, which is identical in each country, is formalized as in Example 2.1 of Boldrin and Rustichini (1994)). There are two firms, one for each

sector. The firm in the first sector produces a consumption good with two inputs, capital and labor, according to a production function that include externalities from capital, $\hat{F}^1(k^1, \tilde{l}^1, k)$. The externality is assumed to be a labor-augmenting technological progress, i.e. $\hat{F}^1(k^1, \tilde{l}^1, k) = F^1(k^1, k^\eta \tilde{l}^1)$ with $0 \leq \eta < 1$. Let $l^1 = k^\eta \tilde{l}^1$ be the "effective" labor force and assume that $F^1$ is a Cobb-Douglas production function then $F^1(k^1, l^1) = (l^1)^\alpha (k^1)^{1-\alpha}$ with $\alpha \in (0,1)$ where $l^1, k^1$ are the amount of capital and "effective" labor used by the firm of the consumption sector.
In a decentralized economy, the firm maximizes profit

$$Max\ p_t^1\ F^1(k_{t-1}^1, l_t^1) - p_{t-1}^2 k_{t-1}^1 - w_t l_t^1$$

where $p_t^1$ is the price of the consumption good at period $t$, $p_{t-1}^2$ is the price of the capital good at period $t-1$ and $w_t$ the price of labor at period $t$. The optimal production plan satisfies the first order conditions

$$p_t^1 \frac{\partial F^1}{\partial k} = p_{t-1}^2$$

$$p_t^1 \frac{\partial F^1}{\partial l} = w_t$$

In the second sector, the externality is also a labor-augmenting technological progress so it can be treated as above. The representative firm produces a capital good according to a Leontief function $F^2(k^2, l^2) = Min\ (l^2, \frac{k^2}{\gamma})$ with $\gamma \in (0, \alpha)$. The optimal production plan for this firm is then

$$l_t^2 = \frac{k_{t-1}^2}{\gamma}$$

An agent $i$ is characterized by his citizenship, his instantaneous preferences, his labor endowments and his claims of the initial stock of capital. We note this by $(H, u_i, e_i, \theta_i)$. In each period consumers provide inelastically a constant amount of labor $e_i$. We will denote $\sum_{i \in H} e_i = l_H$ the labor supply in country $H$. A model in which the amount of labor provided is endogenously determined could be analyzed but at a much higher cost. At the beginning of the economy, each agent $i$ of country $H$ is endowed with a fixed initial stock $k_{i,-1}$ of capital. With no loss of generality we may note $k_{i,-1} = \theta_i k_{H,-1}$ where $\sum_{i \in H} \theta_i = 1$ and $k_{H,-1}$ is the total initial stock of capital in the economy $H$. Note that if all consumers in country $H$ of size $N_H$ are identical then $e_i = \frac{l_H}{N_H}$ and $\theta_i = \frac{1}{N_H}$. Consumer's preferences are characterized by a discounted utility function of the form

$$U^i(x^i) = \sum_{t=0}^{\infty} \delta^t \quad u_i(x_{it})$$

where $x_{it}$ is the consumption of agent $i$ at time $t$ and $x^i$ is its intertemporal consumption stream. In order to ensure existence of the interior steady state we assume $\delta > \gamma$. The instantaneous utility function fulfills the Inada condition

$$\lim_{x_{it} \to 0} u_i'(x_{it}) = +\infty.$$

In a decentralized economy, an agent $i$ maximizes his utility function subject to a single budget constraint

$$\sum_{t=0}^{\infty} p_t^1 x_{it} = \sum_{t=0}^{\infty} w_t e_i + p_{-1}^2 \theta_i k_{H,-1} \quad \text{with } i \in H$$

A country $H$ is then specified by the size of the population, $N_H$, and the set of its citizens $(H, u_i, e_i, \theta_i)_{i \in H}$. We consider three types of market organization, corresponding to three levels of market integration. Without lack of generality we assume that there are only two countries, i.e. $H = A$ or $B$.

**Scenario 1: Autarchy** *All factors and goods are traded on domestic markets. The economy is denoted* $(A, B)$.
**Scenario 2: Immobile labor** *The capital good and the consumption good are traded on a common market while labor is immobile. The economy is denoted* $(A \cup B)_K$.
**Scenario 3: Complete integration** *All goods and factors are traded on a common market. The economy is denoted* $(A \cup B)$.

# 3 Equilibria and steady states

In this section we first define the competitive equilibrium and then compute the steady states values in the various scenario.

## 3.1 The competitive equilibrium

In the definition of a competitive equilibrium both the externality and the constraints due to the assumptions on the level of integration should be taken into account. In order to encompass all the cases in a unique formalism, the flows in labor, capital and consumption from country $A$ to country $B$ are introduced, and noted $(\tau_t^l, \tau_t^k, \tau_t^x)$. The cases in which none, one or both factors are immobile are obtained by imposing the constraint $\tau_t^j = 0, j = l, k$. A similar remark hold for the case in which the consumption good can only be traded on domestic markets by letting $\tau_t^x = 0$. Finally, the extent of the externality need also to be specified. When $\tau_t^{ex} = 0$ the externality is produced by the domestic amount of capital while when $\tau_t^{ex} = 1$ the externality is worldwide. Clearly, when the countries trade capital on a common market, it may be assumed that the externality is related to the total amount of capital in the world $k_t^\eta$, a proxy for the available knowledge. Let $l_H = \sum_{i \in H} e_i$ the available labor supply in country $H$. A competitive equilibrium can be defined as a sequence satisfying the following definition.

**Definition 1** *A competitive equilibrium is a sequence of prices* $(p_t^1, p_t^2, w_t)_{t=0}^\infty$ *with* $p_{-1}^2 = 1$ *such that markets clear for every* $t \geq 0$*:*

- $l_{At}^1 + l_{At}^2 + \tau_t^l = l_A z_{At}^\eta$
- $l_{Bt}^1 + l_{Bt}^2 - \tau_t^l = l_B z_{Bt}^\eta$
- $z_{At} = k_{A,t-1} + \tau_t^{ex} k_{B,t-1}$
- $z_{Bt} = k_{B,t-1} + \tau_t^{ex} k_{A,t-1}$
- $k_{At}^1 + k_{At}^2 + \tau_t^k = F^2(k_{A,t-1}^2, l_{At}^2)$
- $k_{Bt}^1 + k_{Bt}^2 - \tau_t^k = F^2(k_{B,t-1}^2, l_{Bt}^2)$
- $x_{At} + \tau_t^x = F^1(k_{A,t-1}^1, l_{At}^1)$
- $x_{Bt} - \tau_t^x = F^1(k_{B,t-1}^1, l_{Bt}^1)$
- $k_{A,-1}^1 + k_{A,-1}^2 + \tau_{-1}^k = k_{A,-1}$ *with* $k_{A,-1}$ *given*
- $k_{B,-1}^1 + k_{B,-1}^2 - \tau_{-1}^k = k_{B,-1}$ *with* $k_{B,-1}$ *given*

*where*

- $(\tau_t^l, \tau_t^k, \tau_t^x)$ *are the flows in labor, capital and consumption from country* $A$ *to country* $B$.
- $x_{Ht} = \sum_{i \in H} x_{it}$ *and* $x_{it}$ *is a solution to the individual maximization program of agent* $i$ *for* $(p_t^1, p_t^2, w_t)_{t=0}^{\infty}$.
- $(k_{Ht}^j, l_{Ht}^j)$ *is a solution to profit maximization for firm* $j$, $j = 1, 2$ *in country* $H$ *for* $(p_t^1, p_t^2, w_t)_{t=0}^{\infty}$.

In the sequel of the paper we will analyze the competitive equilibria obtained in the various scenarios. First, we will focus on the stationary competitive equilibria. Then, the local determinacy properties of these will be investigated. The strategy of proof adopted is usual in general equilibrium theory. Indeed, as was recognized by Kehoe, Levine and Romer (1992), every competitive equilibrium obtained in a decentralized economy is a Pseudo-Pareto optimum in the sense that is the solution to the maximization of a social welfare function subject to the constraints (see Ghiglino (2002) for some applications of this method). The social welfare function can be considered as the objective of a constrained worldwide planner. The dynamics of the model may then obtained from the analysis of the Euler equations. The novelty is that only Pseudo-Optimal (or constrained optimal) allocations are considered as the first welfare theorem does not necessary hold. The method has been applied in Ghiglino and Olszak (2005).

## 3.2 The planner's optimum

The Pseudo-Pareto optima (PPO) are the set of solutions to the planner's problem, obtained as $\mu \in [0, 1]^{N-1}$ with $\sum_{i=1}^{N-1} \mu_i \leq 1$ where $N = N_A + N_B$:

$$
\begin{array}{ll}
Max & \sum_{i=1}^{N-1}(\mu_i \sum_{t=0}^{\infty} \delta^t \ u_i(x_{it})) + (1-\sum_{i=1}^{N-1}\mu_i)\sum_{t=0}^{\infty}\delta^t \ u_N(x_{Nt}) \\
s.t & \\
& \sum_{i\in A} x_{it} + \tau_t^x = F^1(k^1_{A,t-1}, l^1_{At}) \text{ for all } t \\
& \sum_{i\in B} x_{it} - \tau_t^x = F^1(k^1_{B,t-1}, l^1_{Bt}) \text{ for all } t \\
& k^1_{At} + k^2_{At} + \tau_t^k = F^2(k^2_{A,t-1}, l^2_{At}) \text{ for all } t \\
& k^1_{Bt} + k^2_{Bt} - \tau_t^k = F^2(k^2_{B,t-1}, l^2_{Bt}) \text{ for all } t \\
& l^1_{At} + l^2_{At} + \tau_t^l = l_A z^\eta_{At} \text{ for all } t \\
& l^1_{Bt} + l^2_{Bt} - \tau_t^l = l_B z^\eta_{Bt} \text{ for all } t.
\end{array}
$$

together with the side conditions

$$
\begin{array}{rcl}
z_{At} & = & k_{A,t-1} + \tau_t^{ex} k_{B,t-1} \\
& & \text{and} \\
z_{Bt} & = & k_{B,t-1} + \tau_t^{ex} k_{A,t-1}
\end{array}
$$

The set of PPO is obtained when $\mu$ spans $[0,1]^{N-1}$ with $\sum_{i=1}^{N-1}\mu_i \leq 1$. A given competitive equilibrium is obtained for a $\mu$ such that the budget constraints of all the consumers bind. For the case with no externalities, $\eta = 0$, and no constraints on the flows, the solutions to the above program are interior as soon as $e_i \neq 0$ for $i \in A, B$. As shown in Ghiglino and Olszak-Duquenne (2001) this is a consequence of the Inada condition on preferences and technology.

A first distinction has to be made. If all inputs and the consumption good are traded only on domestic markets, the Pseudo-Pareto program may be decomposed in two separate maximization programs, each of these programs representing a unique country. In each country, there is a common market for all inputs and goods so that the dynamics of this country can be analyzed in a similar way to the case of full integration of countries. We will therefore assume, except when noted, that the consumption good and capital are traded on a common market. The externality is then assumed to be worldwide, i.e. $\tau^{ex} = 1$.

Let $u_\mu$ be a social utility function defined by

$$\begin{aligned} u_\mu(x) &= Max \sum_{i=1}^{N-1} \mu_i u_i(x_{it}) + (1 - \sum_{i=1}^{N-1} \mu_i)\; u_N(x_{Nt}) \\ & \quad s.t \;\; \sum_{i=1}^{N} x_{it} = x \end{aligned}$$

Let $T(k, y, z)$ be the world transformation function defined as the maximum aggregate output in the consumption good for given initial and final values of the total capital stock, $(k, y)$, and externality $(z)$. Note that the solution need to be compatible with the constraints on the flows of inputs across countries and may depend on $l_A$ and $l_B$. $T(k, y, z)$ is then defined as the solution to

$$\begin{aligned} Max \quad & F^1(k_A^1, l_A^1) + F^1(k_B^1, l_B^1) \\ s.t. \quad & \\ & y \leq F^2(k_A^2, l_A^2) + F^2(k_B^2, l_B^2) \\ & k_A^1 + k_A^2 + k_B^1 + k_B^2 = k \\ & l_A^1 + l_A^2 + \tau^l = l_A z^\eta \\ & l_B^1 + l_B^2 - \tau^l = l_B z^\eta \end{aligned}$$

The planner's problem is seen to be equivalent to

$$\begin{aligned} Max \quad & \sum_{t=0}^{\infty} \delta^t u_\mu(T(k_{t-1}, k_t, z_t)) \\ s.t. \quad & k_t = k_{At} + k_{Bt} \leq F^2(k_{A,t-1}, l_A z_t^\eta) + F^2(k_{B,t-1}, l_B z_t^\eta) \\ & k_{A,-1} \text{ and } \; k_{B,-1} \text{ given} \end{aligned}$$

In general, the solution depends on $(z_t)$ and $(k_{A,-1}, k_{B,-1})$. However, there is still a side condition which depends on the type of externality: the condition is $z_t = k_{t-1} = k_{A,t-1} + k_{B,t-1}$ because the externality is worldwide.

In the sequel we use the return function $V : R_+ \times R_+ \times R_+ \rightarrow R$ defined by

$$V(k, y, z)) = u_\mu(T(k, y, z))$$

The function $V$ is concave in $(k, y)$, because $u$ and $T$ are concave. Note that $V$ and $T$ may depend on $(l_A, l_B)$.

Using the return function $V$, the maximization program can be written as

$$\begin{array}{lrl} Max & & \sum_{t=0}^{\infty} \delta^t \; V(k_t, k_{t+1}, z_{t+1}) \\ & s.t. \quad (k_t, k_{t+1}) \in & D_t \\ & & k_{-1} = k_{A,-1} + k_{B,-1} \text{ given} \end{array}$$

where $D_t$ is the set $\{(k_t, k_{t+1})| \; k_{t+1} \leq F^2(k_{A,t}, l_A z_{t+1}^{\eta}) + F^2(k_{B,t}, l_B z_{t+1}^{\eta}), k_t = k_{A,t} + k_{B,t}, k_{A,t} > 0, k_{B,t} > 0\}$. Furthermore, the externality satisfy $z_t = k_{t-1} = k_{A,t-1} + k_{B,t-1}$.

When both factors are mobile and the consumption good is traded on a world market, the return function can be reduced to a function of only the aggregated variables $(k, y, z)$. Let $V_1(k, y, z) = \partial V(k, y, z)/\partial k$ and $V_2(k, y, z) = \partial V(k, y, z)/\partial y$. It is a standard result that the set of interior Pareto optima are the set of $\{k_t\}_t$ that satisfies the transversality condition $\lim_{t \longrightarrow \infty} \delta^t V_1(k_t, k_{t+1}, k_{t-1})k_t = 0$ and are solutions to the system

$$V_2(k_t, k_{t+1}, k_t) + \delta V_1(k_{t+1}, k_{t+2}, k_{t+1}) = 0 \quad \forall t$$

## 3.3 The steady state values

The steady state values of consumption and capital, both worldwide and countrywide, are affected by the level of market integration. When both factors are mobile and the consumption good is traded on a world market, the return function can be reduced to a function of only the aggregated variables $(k, y, z)$. At a steady state $k_t = k^*$ for every $t \geq 0$. The capital $k^*$ is implicitly defined by the equation

$$V_2(k^*, k^*, k^*) + \delta V_1(k^*, k^*, k^*) = 0$$

In models with a unique consumption sector, aggregate steady state variables depend only on the technology. Indeed, using the definition of the return function, the Euler condition can be written as

$$T_2(k^*, k^*, k^*) + \delta T_1(k^*, k^*, k^*) = 0 \tag{1}$$

where $T_1(x, y, z) = \partial T(x, y, z)/\partial x$ and $T_2(x, y, z) = \partial T(x, y, z)/\partial y$. The solution to equation (1) is the aggregate capital steady state. Furthermore the individual consumption allocations depend on the wealth of the agents but not on the instantaneous utility function. The result is formalized in the following lemma.

**Lemma 1** ***Full integration.*** ***i)*** *Consider a fully integrated economy $(A \cup B)$ where $l = l_A + l_B$. Let $k^*$ and $x^*$ be the steady state values of worldwide capital and consumption. Then*

$$k^* = \left[ l \frac{(1-\alpha)(\gamma-\delta)}{\gamma - \alpha - \delta(1-\alpha)} \right]^{\frac{1}{1-\eta}}$$

*and*

$$x^* = k^* (l k^{*\eta - 1} - 1)^{\alpha} (1-\gamma)^{1-\alpha}$$

***ii)*** *Let $x_i^*$ the consumptions of agent $i$ in the economy $(A \cup B)$. Then*

$$x_i{}^* = \frac{x^*}{1-\gamma} \left[ (\delta(1-\alpha) + \alpha - \gamma) e_i + (1-\delta)(1-\alpha)\widetilde{\theta}_i \right]$$

*where $\widetilde{\theta}_i = \theta_i \left(\frac{l_H}{l}\right)^{\frac{1}{1-\eta}}$*

**Proof:** Note that it is assumed that $\gamma < \delta$. For the details see the Appendix. Q.E.D.

Note that both $k^*$ and $x^*$ depend on the total labor supply according to the factor $l^{\frac{1}{1-\eta}}$. On the other hand the per-capital values depend on $l^{\frac{\eta}{1-\eta}}$. Furthermore, Lemma 1 implies that for given technology parameters and discount factor, the steady state value of consumption in each country depends linearly on holdings in initial capital and labor endowments. This also means that there is a linear manifold of $(\theta, e)$ associated to each equilibrium allocation. This fact is reminiscent of the fibration of the equilibrium manifold in smooth general equilibrium economies.

When labor is immobile, the return function can still be written as a function of the aggregate variables keeping in mind that it depends on $l_A$ and $l_B$. The steady state is defined by

$$T_2^{k-mobile}(k,k,k) + \delta T_1^{k-mobile}(k,k,k) = T_2^A(k_A,k_A,k) + \delta T_1^A(k_A,k_A,k) = 0$$

The following lemma gives the result.

**Lemma 2** ***Immobile labor. i)*** *Assume that capital is mobile but labor is not. Let $k_A^*$ and $k_B^*$ be the steady state values of capital in country A and B. Then $k_A^*$ and $k_B^*$ are the solutions to*

$$-\left[\alpha + \gamma(1-\alpha)(\frac{l_A(k_A+k_B)^\eta - k_A}{k_A - \gamma k_A})\right] + \delta(1-\alpha)\left(\frac{l_A(k_A+k_B)^\eta - k_A}{k_A - \gamma k_A}\right) = 0$$

*and*

$$\frac{l_A(k_A+k_B)^\eta - k_A}{k_A - \gamma k_A} = \frac{l_B(k_A+k_B)^\eta - ((k_A+k_B) - k_A)}{((k_A+k_B) - k_A) - \gamma((k_A+k_B) - k_A)}$$

***ii)*** *Assume also that labor endowments are equal, i.e. $l_A = l_B = l/2$. Then*

*the worldwide steady state capital $k^{*(k)}$ is obtained for $k_A^* = k_B^* = k^*/2$ and*

$$k^{*(k)} = \left[l\frac{(1-\alpha)(\gamma-\delta)}{\gamma - \alpha - \delta(1-\alpha)}\right]^{\frac{1}{1-\eta}} = k^*$$

*Furthermore, the steady state consumptions are identical to those obtained in the case of complete integration (Lemma 1).*

**Proof:** See the Appendix. Q.E.D.

Note that according to the previous lemma, the steady state in the economy with immobile labor is identical to the one obtained with mobile factors.

Finally, if both countries are in autarchy each country evolve according to its own Euler equation. Indeed, in this case it may be assumed that both factors are immobile and that the externality is only domestic. At the steady state we have the two equations

$$T_2^H(k_H^*, k_H^*, k_H^*) + \delta T_1^H(k_H^*, k_H^*, k_H^*) = 0, \ H = A, B.$$

The result is contained in the following result.

**Lemma 3** ***Autarchy.*** ***i)*** *Assume that countries are in a regime of autarchy, i.e. the economy is* $(A, B)$*. Let* $k_H^*$ *be the steady state value of capital in country* $H$*. Then*

$$k_H^* = \left[ l_H \frac{(1-\alpha)(\gamma-\delta)}{\gamma-\alpha-\delta(1-\alpha)} \right]^{\frac{1}{1-\eta}}$$

***ii)*** *If* $l_A = l_B = \frac{l}{2}$ *then* $k_A^* = k_B^* = \frac{k^*}{2}$ *and*

$$x_H^* = k_H^*(l_H k_A^{*\eta-1} - 1)^\alpha (1-\gamma)^{1-\alpha}$$

**Proof:** See the Appendix. Q.E.D.

Note that $k_A^* + k_B^*$ is not equal to the aggregate capital obtained with mobile factors, this because of the externality.

The above Lemmas show that when both factors are traded on a given common market, the steady state value of capital depends on the aggregate values of the endowments in the primary commodities available on the relevant market. This result is because the steady state does not depend on the return function, and therefore is independent of the welfare weights and the individual preferences and endowments. The same result is true if labor is immobile but both countries are endowed with the same quantity. In more general models this is not true. In particular, when there are two consumption goods the steady state values of aggregate consumption depend on the welfare weights and therefore on the heterogeneity in preferences and endowments across countries. As opposed to aggregate variables, the steady state consumptions in each country do depend on individual characteristics through the welfare weights (note that at a steady state these are easy to compute).

# 4 Local Indeterminacy in the global economy

In the present section we investigate the local determinacy properties of the steady states as a function of the distribution of endowments in inputs. Near the steady state the behavior of the dynamic system is equivalent to the behavior of the linearized system. The dynamic properties of the steady state are then related to the two eigenvalues of the matrix associated to the linearized system. In particular, the stability and the local determinacy properties of the steady state depends on how the modulus of the eigenvalues compare to one.

When capital is mobile, the return function has only aggregate quantities as arguments, although the function itself may depend on the individual labor endowments if labor is immobile. Let the function $f$ be defined by

$$\begin{aligned} f(k_t, k_{t+1}, k_{t+2}) &= V_2(k_t, k_{t+1}, k_t) + \delta V_1(k_{t+1}, k_{t+2}, k_{t+1}) \\ &= u'(T(k_t, k_{t+1}, k_t))T_2(k_t, k_{t+1}, k_t) + \\ &\quad \delta u'(T(k_{t+1}, k_{t+2}, k_{t+1}))T_1(k_{t+1}, k_{t+2}, k_{t+1}) \end{aligned}$$

The Euler equation

$$f(k_t, k_{t+1}, k_{t+2}) = 0$$

can be linearized at the steady state $(k^*, k^*, k^*)$ to give

$$\frac{\partial f}{\partial k_{t+2}}|_{(k^*,k^*,k^*)}(k_{t+2}-k^*)+\frac{\partial f}{\partial k_{t+1}}|_{(k^*,k^*,k^*)}(k_{t+1}-k^*)+\frac{\partial f}{\partial k_t}|_{(k^*,k^*,k^*)}(k_t-k^*) = 0$$

This equation describes the dynamic behavior of $\{k_t\}_t$ near $k^*$. The resolution of the characteristic equation

$$\frac{\partial f}{\partial k_{t+2}}|_{(k^*,k^*,k^*)}p^2 + \frac{\partial f}{\partial k_{t+1}}|_{(k^*,k^*,k^*)}p + \frac{\partial f}{\partial k_t}|_{(k^*,k^*,k^*)} = 0$$

for $p$ gives the eigenvalues associated to the dynamical system. In a two-sector model with only one consumption good, for a given technology and size of the economy the set of steady state aggregate values is uniquely determined and does not depend on the characteristics of the consumers. However, the stability properties of the steady states do depend on the social utility function. The following notion need to be introduced.

**Definition 2** *Let $u_i$ be the utility function of an agent $i$, $u_i : R_+ \to R$, Then $\rho_i(x_i)$ is the inverse of the absolute risk aversion of agent $i$, $\rho_i(x_i) = -\frac{u_i'(x_i)}{u_i''(x_i)}$ Similarly, let $u_H$ be the social utility function of country $H$, $u_H : R_+ \to R$. Then $\rho_H$ is the inverse of absolute risk aversion, $\rho_H(x_H) = -\frac{u'(x_H)}{u''(x_H)}$ associated to total consumption $x_H$.*

For a given externality, discount factor and technology parameters, the eigenvalues depend on the curvature $\rho_H$ of the social utility function, through the equality $V(k, y, z) = u(T(k, y, z))$. Consequently, the determinacy and stability properties of the steady states depends on the "social" utility function associated to the Pseudo-Pareto program. The following lemma gives necessary and sufficient conditions on the technology parameters and the time discount factor such that the determinacy property of the steady state may be affected by changes in the curvature of the "social" utility function.

**Lemma 4** ***(i) Full integration and autarchy.*** *If*

$$\frac{1}{3} < \alpha < \frac{1}{2} \quad ; \quad \gamma > 1 - 2\alpha \quad ; \quad \eta > \overline{\eta} = \frac{2\alpha + \gamma - 1}{1 - \gamma}$$

*or*

$$\alpha > \frac{1}{2} \quad ; \quad \gamma < 1 - \alpha \quad ; \quad \eta > \overline{\eta}$$

*then there exists $\overline{\delta}$ such that for all $\delta$ in $I_{\delta 1} = ]\overline{\delta}, 1[$, there exists $\rho_c > 0$ such that $p_2(\rho_c) = -1$. There is indeterminacy for $\rho > \rho_c$ and determinate stability otherwise. Furthermore,*

$$\rho_c = \frac{2\delta T_1(T_1(1+\delta) + T_3)}{T_{22} - T_{12} - T_{23} + \delta(T_{11} + T_{13} - T_{12})}$$

*and* $\rho_c$ *is proportional to* $l^{\frac{1}{1-\eta}}$.

***(ii) Immobile labor.*** *For an open set of values of* $\alpha, \gamma, \eta$ *satisfying the conditions given in (i) above there exists* $\overline{\delta}$ *such that for all* $\delta$ *in* $I_{\delta 1} = ]\ \overline{\delta}, 1\ [$, *there exists* $\rho_c > 0$ *such that* $p_2(\rho_c) = -1$. *There is indeterminacy for* $\rho > \rho_c$ *and determinate stability otherwise. Furthermore,*

$$\rho_c = \frac{2\delta T_1(T_1(1+\delta) + \left(\frac{l_A}{l_B} + 1\right) T_3}{T_{22} - T_{12} - T_{23} + \delta(T_{11} + T_{13} - T_{12})}$$

*and* $\rho_c$ *is proportional to* $l^{\frac{1}{1-\eta}}$.

**Proof:** See Appendix. Q.E.D.

The dynamics of the economy with heterogeneous agents can in principle be deduced from the dynamics of the Pseudo-Pareto Optima (PPO) in which the constraints on the size of the market are included. Indeed, once the dynamics of these is known the only thing that remains to be done is to pick the PPO that corresponds to the given distribution of endowments. However, this construction doesn't imply that the local stability and determinacy properties of the steady state can be deduced from the properties of the PPO allocations with the welfare weights fixed at the steady state values (see Ghiglino (2002)). When the welfare weights are continuous functions of the initial conditions, the dynamic and determinacy properties of the general equilibrium model with heterogeneous agents and those of the model with fixed weights are identical. Ghiglino and Olszak-Duquenne (2001) have shown that when there are no externalities, continuity holds. Because of the externality the continuity property cannot be assumed here. However, the analysis of indeterminacy can be pursued to a large extent without this strong property. The fundamental property is that indeterminacy of the solutions to the planner's problem implies the existence of disaggregate economies with indeterminate competitive equilibria. This result is contained in the following Lemma.

**Lemma 5** *A sufficient condition for the existence of local indeterminacy in the general equilibrium world economy with heterogeneous agents is indeterminacy in the model with the welfare weights fixed at their steady state values.*

**Proof:** See Lemma 3 in Ghiglino and Olszak-Duquenne (2005) .

The next Proposition shows that in each market integration scenario there exists economies with indeterminate competitive equilibria.

**Proposition 1** *For each scenario,* $(A, B), (A \cup B)_K$ *and* $(A \cup B)$, *there exists open sets of economies defined by utility functions and parameters* $\alpha$, $\gamma$, $\eta$, $\delta$ *such that local indeterminacy near the competitive steady state occurs.*

**Proof:** Use Lemma 4 to modify the proof of Proposition 1 in Ghiglino and Olszak-Duquenne (2005). Q.E.D.

*Remark:* In Proposition 1, preferences are not bound to belong to some specific class. However, it can easily be shown that the result still holds when preferences are CES. The arguments are similar to Ghiglino and Olszak-Duquenne (2001).

It should be pointed out that Proposition 1 doesn't analyze the effects of redistributions of initial endowments on determinacy. Indeed, this requires to be able to decide whether the steady state of the general disaggregated model is determinate or indeterminate, a much harder task. When the welfare weights are continuous functions of the initial conditions, the local dynamic properties of the general equilibrium model are indeed equivalent to the dynamic properties of the aggregate "optimal growth" model with the welfare weights fixed at the steady state values. In the presence of an externality this is no longer true, so we introduce, as in Definition 3 in Ghiglino and Olszak-Duquenne (2005), the following notions.

**Definition 3** *The steady state is said to be* ***determinate*** *when there is a unique path converging to it for any initial capital taken in a neighborhood of its steady state value. The steady state is said to be* ***locally determinate*** *when there is a unique path converging to the steady state for any initial* $n+1$-*tuple of capital and individual wealth taken in a neighborhood of the steady state values.*

In other words, if $k^*$ is the steady state value of capital and $(w_i^*)_{i=1}^N$ are the associated individual incomes, the steady state is locally determinate if for any $k_0$ close to $k^*$ there doesn't exist another path with $(w_i)_{i=1}^N$ close to $(w_i^*)_{i=1}^N$. We can now state our main result.

**Proposition 2** *For each scenario,* $(A, B), (A \cup B)_K$ *and* $(A \cup B)$*, there exists open sets of economies defined by utility functions and parameters* $\alpha$*,* $\gamma$*,* $\eta$*,* $\delta$ *such that the occurrence local indeterminacy of the competitive steady state depends on the distribution of labor endowments and capital shares.*

**Proof:** Use Lemma 4 to modify the proof of Proposition 1 in Ghiglino and Olszak-Duquenne (2005). Q.E.D.

# 5 Integration, the size of the market and indeterminacy

There are many reasons indicating that joining a common market might be beneficial for all members, at least on the long run. In particular, one could expect that the larger is the market the more stable it is. The present and the next sections investigate this issue from the point of view of the effect of integration on indeterminacy. Market integration may affect the properties of the economic equilibrium through two channels. First, the enlargement of the market brings an increase in the total supply and demand for goods and factors which may change the size of the externality and per-capita consumption. Second, in the presence of heterogeneous agents integration typically affects the level of heterogeneity of the population. In this section we investigate the effect of size on indeterminacy.

In order to focus on the size effect we assume that all agents are identical. Our leading case is that of complete market integration, i.e. starting from full autarchy the reform consists in joining a common market where the produced goods and inputs are traded. We first need a technical Lemma.

**Lemma 6** *Consider an economy $H$ populated with agents with utility function $u$ and let $\eta > 0$. Assume that per-capita endowments $e_H$ and capital shares $\theta_H$ do not change but that the overall population present on the market is increased by a factor $\lambda > 1$. If $\rho_u$ associated to $u$ is a strictly concave function then $\frac{\rho_H}{\rho_c}$ decreases while if $\rho_u$ is a strictly convex function then $\frac{\rho_H}{\rho_c}$ increases.*

**Proof:** Both $\rho_H$ and $\rho_c$ depend on the size of the supply of labor. From the proof of Lemma 4 we know that $\rho_c$ is proportional to $l^{\frac{1}{1-\eta}}$, $\rho_c(\lambda l) = \lambda^{\frac{1}{1-\eta}}\rho_c(l)$. On other hand, Lemma 1 shows that aggregate consumption $x^*$ depend on the total labor supply according to the factor $l^{\frac{1}{1-\eta}}$ and that per-capital consumption depends on $l^{\frac{\eta}{1-\eta}}$. In Ghiglino (2005) it is shown that the value of the curvature at the steady state for an economy with $N_H$ agents is given by

$$\rho_H(x^*) = -\sum_{i=1}^{N_H} n_i \frac{u_i'}{u_i''}(x_i^*) = \sum_{i=1}^{N_H} n_i \rho_u(x_i^*)$$

As all agents are identical, the value of $\rho_H$ only depends on the labor supply, for given fundamentals. For an economy with labor supply $\lambda l$, $\rho_H$ is noted $\rho_{\lambda l}$, and $\rho_{\lambda l} = \lambda l \rho_u(\lambda^{\frac{\eta}{1-\eta}} x_i^*)$ where $x_i^*$ is the individual consumption associated to labor supply $l$. By definition, for any strictly concave function $f$ we have that $f(\lambda l) < \lambda f(l)$ for any $\lambda > 1$. Therefore, strict concavity of $\rho_u$ implies that

$$\frac{\rho_{\lambda l}}{\rho_c(\lambda l)} < \frac{\lambda \lambda^{\frac{\eta}{1-\eta}} \rho_l}{\lambda^{\frac{1}{1-\eta}} \rho_c(l)} = \frac{\rho_l}{\rho_c(l)}$$

so that $\frac{\rho_H}{\rho_c}$ is a strictly decreasing function. On the other hand, if $\rho_u$ is a strictly convex function then $\frac{\rho_H}{\rho_c}$ is a strictly increasing function. Finally, whenever $\rho_u$ is linear size has no effect. Q.E.D.

The previous results show that, provided $\rho_u$ is a concave function, the increase in the size of the market increases per capita steady state consumption without introducing indeterminacy. On the other hand if $\rho_u$ is a strictly convex function the increase in consumption may occur at the cost of an increase in fragility due to indeterminacy. This result is extended the case of market integration in the following two Propositions.

**Proposition 3** *In the class of economies with $u_A = u_B = u$, $e_A = e_B$, $\rho_u$ a concave function and $\eta > 0$ there does not exist open sets of preferences, endowments and technology parameters such that country A* ***and/or*** *country B are determinate while the integrated economy $A \cup B$ is indeterminate. The same holds for the integrated economy $(A \cup B)_K$*

**Proof:** From Lemma 4 we know that for parameters taken in a suitable open set the steady state is indeterminate for $\rho > \rho_c$ and determinate and stable for $\rho < \rho_c$. Note that in the other cases changes in $\rho$ leaves the determinacy properties of the economy unaffected. As agents are all identical Lemma 6 can be applied. Indeed, market integration can be viewed as the growth of country A or country B to a country of size $l_A + l_B$. This means that

$$\frac{\rho_{A\cup B}}{\rho_c^{A\cup B}} < \frac{\rho_A}{\rho_c^A} \text{ and/or } \frac{\rho_{A\cup B}}{\rho_c^{A\cup B}} < \frac{\rho_B}{\rho_c^B}$$

and the result follows. Q.E.D.

**Proposition 4** *In the class of economies with $u_A = u_B = u$, $e_A = e_B$ and $\eta > 0$ provided $\rho$ is a convex function there does not exist open sets of preferences, endowments and technology parameters such that country A* ***and/or*** *country B are indeterminate while the integrated economy $A \cup B$ is determinate. The same holds for the integrated economy $(A \cup B)_K$*

**Proof:** As previous Proposition. Q.E.D.

Two cases are not covered by Proposition 3 and Proposition 4. When both countries are determinate but $\rho_u$ is convex an increase in the size tends to increase indeterminacy as $\frac{\rho_H}{\rho_c}$ increases with size. It is then in general impossible to conclude. The same lack of conclusion affects the case with two indeterminate countries with a concave $\rho_u$ integrating a common market. In both cases the outcome depends on the exact circumstances of the integration and will be analyzed in the next section.

An implication of Proposition 3 and Proposition 4 is that the usual axioms on preferences don't restrict the nature of the effect of size on indeterminacy.

The reason is that the characterization involves third and other high order derivatives of the utility functions. Standard assumptions on preferences do not put any limitation on these and empirical data is also lacking (see Gollier (2001)).

The following result concern preferences with the HARA property. Note that this class include most of the commonly used specifications, as the CARA and CRRA.

**Lemma 7** ***HARA preferences.*** *Assume that individual preferences can be represented by a utility function of the HARA class, i.e.*

$$v(x) = \frac{1-\gamma}{\gamma}\left(\frac{ax}{1-\gamma} + b\right)^{\gamma}$$

*with $a, b$ and $\gamma$ as parameters. Then market integration plays no role on the occurrence of indeterminacy.*

**Proof:** Indeed, HARA utility functions are characterized by $u'(x)u'''(x)(u''(x))^{-2} = k$ with $k > 0$, so that $(R^{-1}(x))''=(u'(x)u''(x)^{-1})''=$ $=(((u''(x))^2 - (u'(x)u'''(x))(u''(x))^{-2})' = (-u'(x)u'''(x)(u''(x))^2)' = 0$ (see Caroll and Kimball (1996)). Q.E.D.

The result shows that in many usual situations, as the one with CES preferences, indeterminacy and the associated fragility of expectations, is not likely to be increased by market integration.

# 6 Integration, heterogeneity and indeterminacy

In the previous section we analyze how the increase in the size of the market associated to integration affects indeterminacy. In this section we investigate the effect of market integration in the presence of heterogeneity. It turns out that the results are not affected much by heterogeneity and Proposition 3 and Proposition 4 are slightly modified. On the other hand, in the cases not covered by these results heterogeneity may matter significantly.

There are several types of heterogeneity. We start assuming that all agents have the same preferences and that within each country agents are homogeneous. This implies that within a country $H$ agents have the same capital shares $\theta_i = \frac{1}{N_H}$ and that heterogeneity is summarized by $e_A \neq e_B$. There are two possible interpretations fur this type of heterogeneity. On the one hand, differences in labor endowments may be a proxy for differences in the "quality" of the labor supplied. On the other hand, the differences may represent differences in the average labor supply, which can be interpreted as differences in the rate of employment. We first state the equivalent of Proposition 3 and Proposition 4.

**Proposition 5** *Assume that the economy is populated with agents having the same preferences, $u_A = u_B = u$, but general endowment distribution. Assume that there is an externality, i.e. $\eta > 0$, and that $\rho$ is a concave function. Then market integration increases determinacy in the sense of Proposition 3. If $\rho$ is a convex function then market integration increases indeterminacy in the sense of Proposition 4.*

**Proof:** Use Lemma 6 and modify Proposition 3 and Proposition 4. Q.E.D.

The two propositions obtained without heterogeneity remain true when there is heterogeneity both within each country and across countries provided that all agents have the same preferences. However, this does not preclude a role to heterogeneity in the situations which are not covered by Proposition 5. For these the outcome depends on the exact circumstance.

In fact what renders the role of heterogeneity interesting is the possibility to accompany market integration with a redistributive policy in order to achieve or preserve determinacy. The next result characterizes the role of heterogeneity when both countries are determinate and $\rho_u$ is convex.

**Proposition 6** *In the class of heterogeneous economies populated with agents having the same preferences, $u_A = u_B = u$ in which $\rho$ is a strictly convex function and $\eta > 0$ there exist open sets of preferences, endowments and technology parameters such that country $A$ **and** country $B$ are determinate while the integrated economy $A \cup B$ is indeterminate without redistribution and determinate for some open set of redistributions. The same holds for the integrated economy $(A \cup B)_K$*

**Proof:** From Lemma 4 we know that for parameters taken in a suitable open set, the steady state of country $H$ is indeterminate for $\rho_H > \rho_c$ and determinate and stable for $\rho_H < \rho_c$. Note that as soon as the parameters are chosen outside the sets defined in Lemma 4 changes in $\rho$ leaves the determinacy properties of the economy unaffected. Determinacy in A and B implies that $\rho_A < \rho_c(l_A)$ and $\rho_B < \rho_c(l_B)$. As $\rho_c$ is proportional to $l^{\frac{1}{1-\eta}}$ and does not depend on preferences, we have $\rho_c(l_A + l_B) = (\frac{l_A+l_B}{l_A})^{\frac{1}{1-\eta}}\rho_c(l_A) = (\frac{l_A+l_B}{l_B})^{\frac{1}{1-\eta}}\rho_c(l_B)$. On the other hand, in Ghiglino (2005) it is shown that the value of the curvature at the steady state for an economy with $N$ agents with identical utility function $u$ is given by

$$\rho_H(x^*) = \sum_{i=1}^{N_H} n_i \rho_u(x_i^*)$$

where $\rho_u(x_i^*)$ is related to individuals $i$ in country $H$. This expression implies

$$\begin{aligned}
\rho_{A\cup B} &= l_A\rho_u(x_{i\in A}(l_A + l_B)) + l_B\rho_u(x_{i\in B}(l_A + l_B)) = \\
&= l_A\rho_u(x_{i\in A}(\frac{l_A + l_B}{l_A}l_A)) + l_B\rho_u(x_{i\in B}(\frac{l_A + l_B}{l_B}l_B)) = \\
&= l_A\rho_u(\left(\frac{l_A + l_B}{l_A}\right)^{\frac{\eta}{1-\eta}} x_{i\in A}(l_A)) + l_B\rho_u(\left(\frac{l_A + l_B}{l_B}\right)^{\frac{\eta}{1-\eta}} x_{i\in B}(l_B))
\end{aligned}$$

Determinacy of $(A \cup B)$ is obtained if and only if $\rho_{A\cup B} \leq \rho_c(l_A + l_B)$. Then the condition for determinacy is

$$\begin{aligned}
& l_A\rho_u(\left(\frac{l_A + l_B}{l_A}\right)^{\frac{\eta}{1-\eta}} x_{i\in A}(l_A)) + l_B\rho_u(\left(\frac{l_A + l_B}{l_B}\right)^{\frac{\eta}{1-\eta}} x_{i\in B}(l_B)) \\
\leq\ & (\frac{l_A + l_B}{l_A})^{\frac{1}{1-\eta}}\rho_c(l_A)
\end{aligned}$$

Convexity of $\rho_u$ implies that $\rho_u(\left(\frac{l_A+l_B}{l_H}\right)^{\frac{\eta}{1-\eta}} x_i(l_H)) > \left(\frac{l_A+l_B}{l_H}\right)^{\frac{\eta}{1-\eta}} \rho_u(x_i(l_H))$ while determinacy previous to the reform implied $\rho_A < \rho_c(l_A)$ and $\rho_B < \rho_c(l_B)$. As these inequalities do not provide any sure implication it is clear that open sets of parameters, endowment distributions $e_A$ and $e_B$ and redistributions leading to the situation described in the Proposition can be found. This proves the first part of the claim. Q.E.D.

The intuition behind the result is that due to market integration $\frac{\rho_H}{\rho_c}$ increases so that indeterminacy becomes more likely in the large economy. However, as the value of $\rho_c$ only depends on total labor supply $l_A + l_B$ while $\rho_H$ depends on the distribution of total consumption across the consumers, whether or not the size increase is enough to change the determinacy properties depends on the wealth distribution. We also have the symmetrical result.

**Proposition 7** *In the class of economies populated with agents having the same preferences, $u_A = u_B = u$, in which $\rho$ is a strictly concave function and $\eta > 0$ there exist open sets of preferences, endowments and technology parameters such that country A **and** country B are indeterminate while the integrated economy $A \cup B$ is indeterminate without redistribution and determinate for some open set of redistributions. The same holds for the integrated economy $(A \cup B)_K$*

**Proof:** Similar to Proposition 6. Q.E.D.

So far we have assumed that within each country agents are identical and that preferences do not vary across countries. The results in Proposition 5 are slightly modified by preference heterogeneity in the sense that they apply only to situations in which both countries are determinate or indeterminate.

**Proposition 8** *Let $\eta > 0$. In the class of economies with $\rho_{u,A}$ and $\rho_{u,B}$ concave functions there does not exist open sets of preferences, endowments and technology parameters such that country A **and** country B are determinate while the integrated economy $A \cup B$ is indeterminate. On the other hand if $\rho$ is a convex function there does not exist open sets of preferences, endowments and technology parameters such that country A **and** country B are indeterminate while the integrated economy $A \cup B$ is determinate. The same results hold for the integrated economy $(A \cup B)_K$.*

**Proof:** Similar to Proposition 5. Q.E.D.

Preference heterogeneity gives a role to redistribution even in the hybrid situations in which one of the countries is determinate and the other indeterminate. Assume that the two countries are homogenous. Then both the relative sizes of the countries and the concavity properties of $\rho_{u,A}$ and $\rho_{u,B}$ matter.

**Proposition 9** *In the class of heterogeneous economies populated with agents having the different preferences, $\rho_{u,A}$ and $\rho_{u,B}$ strictly concave functions and $\eta > 0$ there exist open sets of preferences, endowments and technology parameters such that country $A$ is determinate, country $B$ is indeterminate while the integrated economy $A \cup B$ is determinate. On the other hand if $\rho_{u,A}$ and $\rho_{u,B}$ are strictly convex functions the results holds with the integrated economy $A \cup B$ being indeterminate. The same results holds when the integrated economy is $(A \cup B)_K$*

**Proof:** Similar to proof of Proposition 6. Q.E.D.

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# 8 Appendix

## 8.1 Proof of Lemma 1, Lemma 2 and Lemma 3

### 8.1.1 The return function for the different levels of integration

The Pseudo-Pareto optima were obtained as the solution to the maximization of the discounted sum of the values taken by the return function at each date, taking into account the feasibility constraints. As is well known, the dynamics of the economy may be analyzed through the Euler equations obtained from this program. Note that the local dynamic and determinacy properties of equilibrium paths near the steady state depend on the first and second order derivatives of the return function.

**i) Full integration**

As there is full integration, all factors and the consumption good are traded on a common market. Since there are no constraints on the flows, the variables $(\tau_t^l, \tau_t^k, \tau_t^x)$ are free and endogenously determined and $\tau_t^{ex} = 1$. As only aggregate variables matter here, the transformation function can be written as $T(k, y, z)$ and only depends on the aggregate labor supply $l_A + l_B = l$. It is given by the solutions to

$$\begin{aligned} Max \quad & F^1(k_A^1, l_A^1) + F^1(k_B^1, l_B^1) \\ s.t. \quad & y \leq F^2(k_A^2, l_A^2) + F^2(k_B^2, l_B^2) \\ & k_A^1 + k_A^2 + k_B^1 + k_B^2 = k \\ & l_A^1 + l_A^2 + l_B^1 + l_B^2 = l z^\eta \end{aligned} \tag{2}$$

Because the two countries have the same technology and these are constant returns, program (2) becomes

$$\begin{aligned} Max \quad & F^1(k^1, l^1) \\ s.t. \quad & y \leq F^2(k^2, l^2) \\ & k^1 + k^2 = k \\ & l^1 + l^2 = l z^\eta \end{aligned}$$

With the specification of production adopted through the paper, the transformation function with full integration is then

$$T^{A \cup B}(k, y, z) = (l z^\eta - y)^\alpha (k - \gamma \; y)^{1-\alpha}$$

so that

$$T_1^{A\cup B}(k,y,z) = (1-\alpha)\left(\frac{lz^\eta - y}{k-\gamma y}\right)^\alpha \tag{3}$$

and

$$T_2^{A\cup B}(k,y,z) = -\left(\frac{lz^\eta - y}{k-\gamma y}\right)^{(\alpha-1)}\left[\alpha + \gamma(1-\alpha)(\frac{lz^\eta - y}{k-\gamma y})\right] \tag{4}$$

and finally

$$T_3^{A\cup B}(k,y,z) = \alpha\eta l z^{\eta-1}\left(\frac{lz^\eta - y}{k-\gamma y}\right)^{(\alpha-1)} \tag{5}$$

where $T_1(k,y,z) = \dfrac{\partial T(k,y,z)}{\partial k}$, $T_2(k,y,z) = \dfrac{\partial T(k,y,z)}{\partial y}$ and $T_3(k,y,z) = \dfrac{\partial T(k,y,z)}{\partial z}$. Also the return function is a function of the aggregated quantities only. It is defined by

$$V^{A\cup B}(k,y,z) = u(T^{A\cup B}(k,y,z))$$

**ii) The case with mobile capital and immobile labor**

Let $T^H(k_H, y_H, z)$ be the transformation function for country $H$ defined as the solution to

$$\begin{aligned} Max \quad & F^1(k^1, l^1) \\ s.t. \quad & y_H \leq F^2(k^2, l^2) \\ & k^1 + k^2 = k_H \\ & l^1 + l^2 = l_H z^\eta \end{aligned}$$

where it is assumed that the externality $z$ is worldwide. The aggregate transformation function $T(k, y, z)$ can then be defined as the solution to

$$\begin{aligned} Max \quad & T^A(k_A, y_A, z) + T^B(k_B, y_B, z) \\ s.t. \quad & y_A + y_B = y \\ & k_A + k_B = k \end{aligned}$$

or

$$T^{k-mobile}(k, y, z) = Max_{(k_A, y_A)}\ T^A(k_A, y_A, z) + T^B(k - k_A, y - y_A, z)$$

with

$$V^{k-mobile}(k, y, z) = u(T^{k-mobile}(k, y, z))$$

The optimal values of $(k_A, y_A)$ for a given $(k, y)$ are obtained from the first order conditions defining $T(k, y, z)$. These are

$$\begin{aligned} T_1^A(k_A, y_A, z) - T_1^B(k - k_A, y - y_A, z) &= 0 \\ T_2^A(k_A, y_A, z) - T_2^B(k - k_A, y - y_A, z) &= 0 \end{aligned}$$

However, note that this formalism takes into account the fact that labor is immobile while capital is traded on a common market, i.e. $T^{k-mobile}$ depends on $l_A$ and $l_B$. The envelope theorem implies

$$\begin{aligned} T_1^{k-mobile}(k, y, z) &= T_1^A(k_A, y_A, z) = T_1^B(k - k_A, y - y_A, z) \\ T_2^{k-mobile}(k, y, z) &= T_2^A(k_A, y_A, z) = T_2^B(k - k_A, y - y_A, z) \end{aligned}$$

Furthermore, $T_3^{k-mobile}(k, y, z) = T_3^A(k_B, y_B, z) + T_3^B(k_B, y_B, z)$. Finally, the second order derivatives are $T_{jk}(k, y, z) = T_{jk}^B(k_B, y_B, z)$ for $j$= $1, 2$ and $k = 1, 2, 3$.

The above expressions can be simplified using the specification of production adopted through the paper. The transformation function in each country $j$, $j = A, B$, can be written as

$$T^j(k_j, y_j, z) = (l_j z^\eta - y_j)^\alpha (k_j - \gamma y_j)^{1-\alpha}$$

so that

$$T_1^j(k_j, y_j, z) = (1-\alpha)\left(\frac{l_j z^\eta - y_j}{k_j - \gamma y_j}\right)^\alpha \tag{6}$$

and

$$T_2^j(k_j, y_j, z) = -\left(\frac{l_j z^\eta - y_j}{k_j - \gamma y_j}\right)^{(\alpha-1)} \left[\alpha + \gamma(1-\alpha)(\frac{l_j z^\eta - y_j}{k_j - \gamma y_j})\right] \tag{7}$$

and finally

$$T_3^j(k_j, y_j, z) = \alpha\eta l_j z^{\eta-1}\left(\frac{l_j z^\eta - y_j}{k_j - \gamma y_j}\right)^{(\alpha-1)} \tag{8}$$

Equation (6) implies that

$$(1-\alpha)\left(\frac{l_A z^\eta - y_A}{k_A - \gamma y_A}\right)^\alpha = (1-\alpha)\left(\frac{l_B z^\eta - y_B}{k_B - \gamma y_B}\right)^\alpha$$

or

$$\frac{l_A z^\eta - y_A}{k_A - \gamma y_A} = \frac{l_B z^\eta - y_B}{k_B - \gamma y_B} = \frac{\omega_B z^\eta - (y - y_A)}{(k - k_A) - \gamma(y - y_A)} \tag{9}$$

Condition (7) gives

$$\begin{aligned} & \left(\frac{l_A z^\eta - y_A}{k_A - \gamma y_A}\right)^{(\alpha-1)} \left[\alpha + \gamma(1-\alpha)(\frac{l_A z^\eta - y_A}{k_A - \gamma y_A})\right] \\ = & \left(\frac{l_B z^\eta - y_B}{k_B - \gamma y_B}\right)^{(\alpha-1)} \left[\alpha + \gamma(1-\alpha)(\frac{l_B z^\eta - y_B}{k_B - \gamma y_B})\right] \end{aligned}$$

Note that every solution to the first condition is also a solution to the latter one, which can then be disregarded. Finally, the first condition implies

$$\begin{aligned} T^{k-mobile}(k, y, z) &= Max_{(k_A, y_A)}\ T^A(k_A, y_A, z) + T^B(k - k_A, y - y_A, z) \\ &= \left(\frac{l_A z^\eta - y_A}{k_A - \gamma y_A}\right)(k + \gamma y) \text{ with } (k_A, y_A) \text{ such that} \\ & \quad \frac{l_A z^\eta - y_A}{k_A - \gamma y_A} = \frac{l_B z^\eta - (y - y_A)}{(k - k_A) - \gamma(y - y_A)} \end{aligned}$$

Furthermore, $T_3^{k-mobile}(k,y,z) = T_3^A(k_A,y_A,z) + T_3^B(k_B,y_B,z)$

$= (1+\frac{l_A}{l_B})T_3^B(k_B,y_B,z)$. Indeed,

$$(1-\alpha)\left(\frac{l_A z^\eta - y_A}{k_A - \gamma y_A}\right)^\alpha = (1-\alpha)\left(\frac{l_B z^\eta - y_B}{k_B - \gamma y_B}\right)^\alpha$$

implies that

$$T_3^A(k_A,y_A,z) = \alpha\eta l_A z^{\eta-1}\left(\frac{l_B z^\eta - y_B}{k_B - \gamma y_B}\right)^{(\alpha-1)} = \frac{l_A}{l_B}T_3^B(k_B,y_B,z) \qquad (10)$$

**iii) Autarchy**

When countries evolve in autarchy, all goods and inputs are traded on domestic markets and externalities are only domestic. Furthermore, it may be implicitly assumed that there are no financial flows across countries. In this case, each country evolves separately, the worldwide dynamics being the direct sum of these domestic dynamics. Then, we have

$$V_A^{autarchy}(k_A,y_A,z_A) = u_A(T^A(k_A,y_A,z_A))$$
$$V_B^{autarchy}(k_B,y_B,z_B) = u(T^B(k_B,y_B,z_B))$$

where the domestic transformation functions are as in the previous case. In other words, for country $j$ we have that $T^j(k_j,y_j,z_j)$ is given by the solution to

$$\begin{aligned} & Max\ F^1(k_j^1,l_j^1) \\ & s.t. \\ y_j &= y_j^1 + y_j^2 = F^2(k_j^2,l_j^2) \\ k_j^1 + k_j^2 &= k_j \\ l_j^1 + l_j^2 &= l_j z_j^\eta \end{aligned}$$

As usual we have

$$T^j(k_j,y_j,z_j) = (l_j z_j^\eta - y_j)^\alpha (k_j - \gamma\ y_j)^{1-\alpha}$$

so that

$$T_1^i(k_j, y_j, z_j) = (1-\alpha)\left(\frac{l_j z_j^\eta - y_j}{k_j - \gamma\ y_j}\right)^\alpha \tag{11}$$

and

$$T_2^i(k_j, y_j, z_j) = -\left(\frac{l_j z_j^\eta - y_j}{k_j - \gamma\ y_j}\right)^{(\alpha-1)}\left[\alpha + \gamma(1-\alpha)(\frac{l_j z_j^\eta - y_j}{k_j - \gamma\ y_j})\right] \tag{12}$$

#### 8.1.2 Proof of Lemma 1(i)

In this case the return function can be reduced to a function of only the aggregated variables $(k, y, z)$. It is a standard result that the set of interior Pareto optima are the set of $\{k_t\}_t$ that satisfies the transversality condition $\lim_{t\longrightarrow\infty} \delta^t V_1(k_t, k_{t+1}, k_{t-1})k_t = 0$ and are solutions to the system

$$V_2(k_t, k_{t+1}, k_t) + \delta V_1(k_{t+1}, k_{t+2}, k_{t+1}) = 0 \quad \forall t$$

At a steady state $k_t = k^*$ for every $t \geq 0$. The capital $k^*$ is implicitly defined by the equation

$$V_2(k^*, k^*, k^*) + \delta V_1(k^*, k^*, k^*) = 0$$

Using the expression of the return function obtained in the previous section the equation

$$T_2(k^*, k^*, k^*) + \delta T_1(k^*, k^*, k^*) = 0$$

gives the steady state capital as a function of the discount factor and the technology parameters only

$$k^* = \left[l\frac{(1-\alpha)(\gamma-\delta)}{\gamma - \alpha - \delta(1-\alpha)}\right]^{\frac{1}{1-\eta}}$$

The aggregate consumption $x^*$ can then be obtained

$$x^* = T(k^*, k^*, k^*) = k^*(lk^{*\eta-1} - 1)^\alpha(1-\gamma)^{1-\alpha}$$

### 8.1.3 Proof of Lemma 1(ii).

The first order conditions associated to the individual program for $i \in H$ are

$$\begin{cases} \delta^t u_i'(x_{it}) & = & \xi_i p_t^1 \qquad \forall t \geq 0 \\ \sum_{t=0}^{\infty} p_t^1 x_{it} & = & \sum_{t=0}^{\infty} w_t e_i + \theta_i k_{H,-1} \end{cases}$$

where $\xi_i$ is the Lagrange multiplier associated to the constrained maximization problem. At a steady state, $x_{it} = x_i^*$ and $p_t^1 = \delta^t p_0^1$ where $p_0^1$ is the price of the consumption good in period 0. On the other hand, the budget constraint implies

$$x_i^* = \frac{1-\delta}{p_0^1}\left(\sum_{t=0}^{\infty} w_t e_i + \theta_i k_{H,-1}\right)$$

As the firms are constant returns to scale, it is irrelevant how the production

of output is splitted among the two countries. As labor the markets are fully integrated, wages are not index by the country. On the other hand the share $\theta_i k_{H,-1}$ can be rewritten as $\theta_i \left(\frac{l_H}{l}\right)^{\frac{1}{1-\eta}} k_{-1}$ because $k_{H,-1} = k_{-1}\left(\frac{l_H}{l}\right)^{\frac{1}{1-\eta}}$. Consider the integrated firms $F_1$ and $F_2$. Profit maximization of firm 1 gives

$$w_t = \delta^t p_0^1 \frac{\partial F_1}{\partial l}(k^1, l^1)$$

$$p_{t-1}^2 = \delta^t p_0^1 \frac{\partial F_1}{\partial k}(k^1, l^1)$$

The demand of agent $i$ at the steady state is:

$$x_i^* = e_i \frac{\partial F^1}{\partial l}(k^1, l^1) + (1-\delta)\theta_i \left(\frac{l_H}{l}\right)^{\frac{1}{1-\eta}} k_{-1} \frac{\partial F^1}{\partial k}(k^1, l^1)$$

The values of $k^1$ and $l^1$ are obtained by solving the system

$$\begin{cases} l^1 + l^2 = l k^{\eta} \\ k^1 + k^2 = k = F^2(k^2, l^2) \end{cases}$$

However, since the second sector is characterized by a Leontief technology, at the optimum $l^2 = \frac{k^2}{\gamma}$. Therefore, $k^1 = (1-\gamma)k$ and $l^1 = k^\eta - k$. The result follows.

### 8.1.4 Proof of Lemma 2

The equation

$$T_2^{k-mobile}(k,k,k) + \delta T_1^{k-mobile}(k,k,k) = T_2^A(k_A,k_A,k) + \delta T_1^A(k_A,k_A,k) = 0$$

implies that

$$-\left(\frac{l_A k^\eta - k_A}{k_A - \gamma k_A}\right)^{(\alpha-1)}\left[\alpha + \gamma(1-\alpha)\frac{l_A k^\eta - k_A}{k_A - \gamma k_A}\right] + \delta(1-\alpha)\left(\frac{l_A k^\eta - k_A}{k_A - \gamma k_A}\right)^{\alpha} = 0$$

or

$$-\left[\alpha + \gamma(1-\alpha)\frac{l_A k^\eta - k_A}{k_A - \gamma k_A}\right] + \delta(1-\alpha)\frac{l_A k^\eta - k_A}{k_A - \gamma k_A} = 0 \qquad (13)$$

As we have seen in the previous section (see equation (9)), the envelope theorem also gives

$$\frac{l_A k^\eta - k_A}{k_A - \gamma k_A} = \frac{l_B k^\eta - (k - k_B)}{(k - k_A) - \gamma(k - k_A)}$$

The system is composed of two independent equations and two unknowns, $k$ and $k_A$, and therefore a unique solution is expected to exist. Call this $k^{k*}$ and $k_A^{k*}$. When $l_A = l_B = l/2$ the solution is obtained for $k_A = k_B = k/2$. Straightforward substitution in equation (13) gives

$$\begin{aligned} -\left[\alpha + \gamma(1-\alpha)\left(\frac{l/2(k)^\eta - k_A}{k_A - \gamma k_A}\right)\right] + \delta(1-\alpha)\left(\frac{l/2(k)^\eta - k_A}{k_A - \gamma k_A}\right) &= \\ -\left[\alpha + \gamma(1-\alpha)\left(\frac{l/2k^{\eta-1} - (1/2)}{(1/2) - \gamma(1/2)}\right)\right] + \delta(1-\alpha)\left(\frac{l/2k^{\eta-1} - (1/2)}{(1/2) - \gamma(1/2)}\right) &= 0 \end{aligned}$$

giving

$$k^{k*} = \left[ l \frac{(1-\alpha)(\gamma-\delta)}{\gamma-\alpha-\delta(1-\alpha)} \right]^{\frac{1}{1-\eta}} = k^*$$

Note that in this case, the steady state in the economy with immobile labor is identical to the one obtained with mobile factors.

## 8.2 Proof of Lemma 4

### 8.2.1 Lemma 4(i)

Near the steady state the behavior of the dynamic system is equivalent to the behavior of the linearized system. The dynamic properties of the steady state are then related to the eigenvalues of the matrix associated to the linearized system. In particular, the stability and the local determinacy properties of the steady state depends on how the modulus of the eigenvalues $p_1$ and $p_2$ compare to one. From Ghiglino and Olszak-Duquenne (2005) we know that the eigenvalues associated to the dynamic system expressed in terms of the curvature are

$$p_{1,2} = \frac{1}{2} \left[ -B \pm \sqrt{B^2 - 4C} \right]$$

with

$$B = \frac{\rho(T_{22} + \delta(T_{11} + T_{13})) + \delta T_1((1+\delta)T_1 + T_3)}{\delta(-\delta T_1^2 + \rho T_{12})}$$

$$C = \frac{1}{\delta} \left[ 1 + \frac{-\delta T_1 T_3 + \rho T_{23}}{-\delta T_1^2 + \rho T_{12}} \right]$$

with $T_{ij} = T_{ij}(k^*, k^*, k^*),\ i, j = 1, 2, 3$. Remark that $p_1 = -1$ and $p_2 = -1$ are satisfied simultaneously only if $p_1 = \dfrac{-B + \sqrt{B^2 - 4C}}{2} = p_2 = \dfrac{-B - \sqrt{B^2 - 4C}}{2} = -1$ (the choice of the sign depends on the convention $\mid p_1 \mid < \mid p_2 \mid$). This implies $\sqrt{B^2 - 4C} = B - 2 = 0$ and $C = 1$. This is a non generic situation.

Now, $p_{1,2} = -1 \Leftrightarrow \pm\sqrt{B^2 - 4C} = -(B-2)$. This implies $B - C = 1$. Using Lemma 2 which gives the values of $B$ and $C$ in terms of $\rho$, we obtain

$$B - C = 1 \Leftrightarrow \frac{(T_{22} - T_{12} - T_{23} + \delta(T_{11} + T_{13}))\rho + \delta T_1(\delta T_1 + 2T_1 + 2T_3)}{\delta(\rho T_{12} + T_1 T_2)} = 1$$

We find that

$$\rho_c = \frac{-2\delta T_1(T_1(1+\delta) + T_3)}{T_{22} - T_{12} - T_{23} + \delta(T_{11} + T_{13} - T_{12})}$$

Using (3), (4) and (5), this can be rewritten as

$$\rho_c = \alpha\delta x^* \frac{Q(\delta)}{f(\delta)}$$

with

$$Q(\delta) = \delta(1-\alpha)(1+\eta) + (1-\alpha) + \eta(\alpha - \gamma)$$
$$f(\delta) = a\delta^2 + b\delta + c$$

and

$$\begin{cases} a = (1-\alpha)(1-2\alpha)(1+\eta) \\ b = -[\alpha(1-\eta)(1-2\alpha) + \gamma(1+\eta)(2-3\alpha)] \\ c = \gamma[\gamma(1+\eta) + \alpha(1-\eta)] \end{cases}$$

The value $\rho_c$ is an acceptable solution for the equation $p_i = -1$ provided it is strictly negative (as it is the ratio of the first derivative $u'$ and the second derivative $u''$ evaluated at the steady-state. Considering the expression for the derivatives of $T$ we see that the numerator does not depends on the size $l$ while the denominator is proportional to $l^{-\frac{1}{1-\eta}}$. The remaining part of the proofs is a straightforward application of Ghiglino and Olszak-Duquenne (2005).

When both factors and the consumption good are traded on domestic markets, there are two disjoints set of Euler equations. Each set corresponds to a unique country. The previous analysis can still be applied to a given country provided all variables are referred to that country (compare equations (11) and (12) to equations (3) and (4)).

#### 8.2.2 Lemma 4(ii)

We have

$$\rho_c = \frac{-2\delta T_1^{(A\cup B)_K}(T_1^{(A\cup B)_K}(1+\delta) + T_3^{(A\cup B)_K})}{T_{22}^{(A\cup B)_K} - T_{12}^{(A\cup B)_K} - T_{23}^{(A\cup B)_K} + \delta(T_{11}^{(A\cup B)_K} + T_{13}^{(A\cup B)_K} - T_{12}^{(A\cup B)_K})}$$

Substituting the derivatives of $T^{(A\cup B)_K}$ computed earlier in (6), (7), (8) and (10), we have

$$\rho_c = \frac{-2\delta T_1^B(T_1^B(1+\delta) + (1+\frac{l_A}{l_B})T_3^B)}{T_{22}^B - T_{12}^B - T_{23}^B + \delta(T_{11}^B + T_{13}^B - T_{12}^B)}$$

The differences with Scenario 1 should be noted. From the Subsection 4.2 is appears that $T_3^{(A\cup B)_K} = (1+\frac{l_A}{l_B})T_3^B$. Furthermore, as it is assumed that labor endowments are identical, at the steady state $k_a = k_b = k/2$. Finally, all the expression involving $T$ and its first and second derivatives have the aggregate value of $z$ as the externality and not $z^b$. The result follows. Q.E.D.