British Combinatorial Newsletter No. 3 (October 2007).

Welcome to the third British Combinatorial Newsletter. Remember this aims to complement the Bulletin (which will remain a slightly formal record) by giving some additional information, such as details of forthcoming meetings, summaries of recent movements of people, visitors, etc: records of “outreach” activities or recent breakthrough results in the subject: it might include a (solved) combinatorial problem and occasional oddities (as in this edition). British Combinatorial Newsletters will be produced around the start of each academic year (when the movements information is most useful) and also at around the time of the Bulletin (in April) to let you know what is coming up over the Summer. They will be archived on the BCB website.

If you have material which you think might be suitable for inclusion, or suggestions as to how the newsletter should evolve, please contact the Editor, David Penman (dbpenman@essex.ac.uk). Needless to say, I reserve control of the content.

Forthcoming standard BCC Meetings.

Open University Winter Combinatorics Meeting.
The 9th OU Winter Combinatorics one-day meeting will take place on Wednesday 30 January 2008. It will take place in the Christodoulou Meeting Room 11 (CMR 11) at the OU. All are welcome. Coffee will be available from 10:15 in Room Q229. Speakers will include Anthony Hilton (Queen Mary), James Hirschfeld (Sussex), Martin Mačaj (Comenius, Bratislava) and Gordon Royle (Western Australia). Details will be available at http://puremaths.open.ac.uk/combin/ in due course. For further information contact Mike Grannell M.J.Grannell@open.ac.uk, or Terry Griggs, T.S.Griggs@open.ac.uk.

Oxford 1-day meeting in Combinatorics.
This is provisionally scheduled to take place on 11 March 2008, but this may yet change. More details should be available from the website http://www2.maths.ox.ac.uk/combinatorics/ in due course. The organiser is Alex Scott (scott@maths.ox.ac.uk).

Two-day Combinatorics Colloquium at QMUL and LSE – May 2008.
There should be a 2-day Combinatorics meeting in London in May 2008, along the lines of the meeting which took place in 2007. However details are yet to be worked out: they will be announced later.

22nd PCC at Warwick, 21-23 July 2008.
This year's PCC is at the University of Warwick (under the auspices of DIMAP, Warwick’s new centre for Discrete Mathematics and its Applications). The organizers are Haris Aziz (Warwick), Manuela Heuer (Open), Emil Vaughan (QMUL) and Simon Griffiths (Cambridge). The website for the conference is http://go.warwick.ac.uk/pcc2008/ and the invited speakers will include Olivier Hudry (Ecole Nationale Supérieure des Télécommunications) and Imre Leader (Cambridge). There is a blog (something to do with modern technology – Ed.) for the conference at http://blogs.warwick.ac.uk/pcc08/, and queries should be sent to pcc@des.warwick.ac.uk As usual, the aim will be to encourage postgraduates to talk about their research for 20 minutes in a non-intimidating atmosphere.
BCC2009 The 2009 BCC (the 22nd) will be at St. Andrews, from Sunday 5-Friday 10 July 2009. The website (at present not very detailed) is http://bcc2009.mcs.st-and.ac.uk/ The Local Organiser is Sophie Huczynska and the other organisers are James Mitchell and Colva Roney-Dougal. Email queries to bcc2009@mcs.st-and.ac.uk

Forthcoming other Meetings.

Criteria for inclusion here include being (a) combinatorial ((very) broadly interpreted) (b) not having already started and (c) having come to the attention of the editor! For more conferences, http://www.maths.qmul.ac.uk/~pjc/bcc/conferences.html or http://www.math.uiuc.edu/~west/meetlist.html are good starting points.

The biannual international symposium on Combinatorial Optimization, CO 2008, will be held at the University of Warwick, 16-19 March 2008. Plenary speakers are Moshe Dror (University of Arizona, USA), Adam Letchford (Lancaster) and Rolf Moehring (TU Berlin, Germany). One page abstracts can be submitted until 19 November 2007. The symposium is organized by Bo Chen, Vladimir Deineko and Arie Koster from Warwick Business School, and is supported by the Centre for Discrete Mathematics and its Applications (DIMAP). More information: http://go.warwick.ac.uk/co2008/

The 3rd Research Workshop on Flexible Network Design will be held at the University of Warwick, July 13 - 16, 2008. The workshop is run by the Centre for Discrete Mathematics and its Applications (DIMAP).

The First International Conference on Combinatorial Physics, CombPhys 1, will take place in Krakow in November 2007. The organisers include Prof. Allan Solomon (Open University). Combinatorial Physics is an emerging area which unites combinatorial and discrete mathematical techniques applied to theoretical physics, especially quantum theory. With this in mind it is an opportune moment to assemble experts working in these fields as well as new entrants to the subject. See http://www.ifj.edu.pl/conf/combphys/ for further details.

(Remember also, in this context, the forthcoming Newton Institute programme on Combinatorics and Statistical Mechanics in January-July 2008: http://www.newton.cam.ac.uk/programmes/CSM/ and a similar program at the Schrodinger Institute, Vienna, around the same time: http://www.mat.univie.ac.at/users/kratt/public_html/esi/ )

Movements

This section is not more accurate than the information given to me – and may well be less so! Some of the events mentioned may have taken place some time ago. Events are in the Mathematics Department (school, faculty,…) unless otherwise indicated.

Bath: Dr. Bernd Sing (formerly OU: aperiodic tilings and sequences, enumerative combinatorics, applications to physics) has taken up a (temporary) Lectureship.
**Bristol:** Dr. Harald Helfgott (combinatorial number theory) has taken up a Lectureship and Prof. Andreas Winter (quantum and classical information theory) a Professorship. Dr. Robert Brignall (permutation patterns, relational structures, partial well order, antichains in partial orders) and Dr. Robert Waters (graph colouring, graph minors, infinite graph theory) are Research Fellows.

**Kent:** Dr. Stephane Launois ($q$-expansions) has taken up a Lectureship.

**Queen Mary, University of London:** Dr. Mark Walters (formerly at Cambridge, probabilistic combinatorics) and Dr. Peter Keevash (formerly at Caltech, Extremal Set systems) have been appointed to Lectureships. Dr. Robert F. Bailey has left to take up a postdoctoral position at Carleton University, Ottawa.

**Oxford:** Oliver Riordan, previously a Royal Society Research Fellow at Cambridge, has (as previously announced) taken up a Professorship at the Mathematical Institute. He is interested in random graphs and other parts of combinatorics. Dr. Stephan Kreutzer (computational logic, logic and graph theory, finite model theory, graph searching games, verification and verification games, model checking, database theory) has taken up a Lectureship in the Computing Laboratory.

**Royal Holloway, University of London:** Dr. Benjamin Klopsch (group theory, additive combinatorics and number theory) has been appointed to a Lectureship.

**Warwick:** Dr. Oded Lachish (coding theory, circuit complexity, sublinear algorithms) has taken up a DIMAP postdoctoral position (based in the Computer Science Department).

**Current and Forthcoming Combinatorial Visitors**

**Queen Mary, University of London:** Prof. Miklós Simonovits (Rényi Institute, Budapest: extremal graph theory and related parts of combinatorics) will be visiting in September-October 2007 and giving some lectures on “*Extremal Graph Theory*”.

**Recent Ph.D theses in Combinatorics.**

Again, not more accurate than the information I receive: “recent” may be ill-defined.


**Crossword.**  
At the Chicago meeting of the AMS on 5-6 October, a special session on Algebraic Coding Theory was held to mark the retirement of combinatorialist Harold N “Thann” Ward from the University of Virginia. Thann is a frequent visitor to the UK, working with Vass Mavron and Tom McDonough at Aberystwyth and with Ray Hill at Salford. As an alternative to presenting a paper in Chicago, Ray compiled a special cryptic crossword in Thann's honour, which was distributed to delegates at the start of the special session. The puzzle is reproduced below for anyone who wishes to have a go at it. The solution will appear in the next issue of the Newsletter, or is available on
request by email to r.hill@salford.ac.uk. Thanks to Alexei Vernitski for help with sorting out formatting issues. And no, we are not guaranteeing a crossword in every edition of the Newsletter!

ACROSS
1 How to get a truncated polyhedron with the minimum of effort (3,7)
6 Over 500 - that's irrational (4)
8 Lear's bull (8)
9 Tailor played bridge in Venice (6)
10 150 from plant cover; maybe 180 from this (4)
11 On it, top men can become supreme (10)
12 Princess can be seen in type of 19 specially studied by 17 (9)
14 Not a penny more for a friend of 17 (5)
17 He appears in the celebrations to mark the three hundredth anniversary of Euler's birth (5)
19 Beatles CD, interspersed with Old English, studied by 17 (5,4)
22 Humpty Dumpty was so strange (3,3,4)
23 Be indebted to sound of clock - to the ear, to the ear (4)
24 Tiny creatures lose nothing in going from England to USA (6)
25 How to help the orienteer, or show that two sets have the same size (4,1,3)
26 One in a hundred - just the opposite, in an earlier time (4)
27 French mathematician in demise of international organization is taken off the list (10)
Down
1 Aeroplane loses direction at Deal (9)
2 Friend of 17, not up to revolutionary five (7)
3 Try to escape or turn if confused (3,3,2)
4 Device of Escher, may lead to conflict in romantic relationships (7,8)
5 17 may be glad that, in his retirement, he may not see another one (6)
6 Vehicles return with beer at the speed of light - that's too clever by half (5-4)
7 Sailors staring madly (7)
13 Easily provoked, biting back in French island (9)
15 Socially unacceptable sauce is found in gloomy place where rough studies are made (6,3)
16 Friend of 17 - did he travel swiftly to get here? (8)
18 Friend of 17 seen in fit of sulks on island (7)
20 American University raised in stone - type of 19 much studied by 17 (7)
21 Prisoner held by Brunhilde, tenuously (6)

Problem.

This is a somewhat counterintuitive fact about vertex-transitive graphs, with scope for new insights in the form of extensions etc. My interest in this area derives largely from recent discussions with Johannes Siemons (UEA).

Let $G$ be a connected finite vertex-transitive graph. (All graphs are finite in what follows, for convenience, though virtually all the questions have natural and interesting analogues for infinite graphs). Let $a_r$ be the number of vertices at distance (exactly) $r$ from a given vertex $v$ in the graph (of course, by vertex-transitivity, this does not depend on the choice of $v$). Thus $a_0$ is 1, $a_1$ the common degree of the vertices, and so forth: and, letting $d$ be the diameter of $G$, $a_r$ is 0 for $r > d$. Our concern here is, so to speak, with what happens in the middle, and in particular the question of whether the sequence $(a_r)$ is unimodal: that is, we have some $t$ such that $a_0 \leq a_1 \leq a_2 \leq ... \leq a_{r-1} \leq a_r \geq a_{r+1} \geq a_{r+2} \geq a_{r+3} \geq ... a_d$.

(It is not unreasonable, intuitively, that the number of vertices at distance $i$ from a given vertex should increase for a while, then gradually decrease: but of course an intuition is not a proof). It is clear that if the graph is not vertex-transitive then (even if we temporarily replace the definition of $a_r$ by the average, over all vertices $v$, of the individual numbers $a_{r,v}$ which are the numbers of vertices at distance $r$ from that particular $v$ in the vertex-transitive case, this is of course equivalent to $a_r$ as defined above), we do not always get a unimodular sequence. For example, take two cliques both with (about) $n/2 - m$ vertices and a path of length (about) $2m$ between the two cliques, where $m$ is small relative to $n$, then clearly most vertices will have many vertices at distance about 1 and (about) $2m$ from them, but very few at any other distances, so the (modified) $a_r$ are not unimodal.

On the other hand, it can be shown that if the graph $G$ is distance-transitive, then $(a_r)$ is log-concave (which implies it is unimodal). The question is, in some sense, how much symmetry structure will guarantee unimodality.
For vertex-transitive graphs, it seems less obvious how one might find “valleys” (i.e. $a_{r-1} > a_r < a_{r+1}$) and certainly I initially reacted by suspecting $(a_r)$ might be unimodal for all vertex-transitive graphs. In fact, however, it is known that even for vertex-transitive graphs, $(a_r)$ need not be unimodal. This appears in a paper by Shearer and Watkins (Counterexamples to Two Conjectures about Distance Sequences. Discrete Mathematics 66 (1987) pp 289-298). It would, however, be interesting to:

(a) know more constructions of vertex-transitive graphs (or graphs with even more symmetry than that) for which $(a_r)$ is not unimodal (be, so to speak, as pathologically non-unimodal as you can – in whatever sense you like!)

(b) know general classes of graphs (other than distance-transitive) for which $(a_r)$ is guaranteed to be unimodal. For example, Johannes tells me that if one takes a Coxeter group with its standard Coxeter generators, then the Cayley graph for this does have unimodal $(a_r)$. Tell us some other interesting classes!

There are some limitations on the pathologies implied by results of Babai and Szegedy (Combinatorics, Probability and Computing 1 (1992) 1-11), and a certain amount is said about these in section 6 of the recent paper http://people.cs.uchicago.edu/~wes/dlocinf.pdf (this is mainly concerned with infinite graphs, but the bibliography looks useful).