Technological Change and Income Distribution Dynamics*

David Andolfatto
Simon Fraser University
dandolfa@sfu.ca

Eric Smith
University of Essex
esmith@essex.ac.uk

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Abstract

This paper explores the link between technological change and the dynamics of employment, production and the distribution of earnings. Technological change not only advances society's collective capability but also changes the relative productivities of its members. This latter effect establishes the likely winners and losers from advances in productive capabilities, provides a mechanism that can generate cyclical fluctuations in output as well as employment, and determines the evolution of the earnings distribution.

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1 Introduction

This paper explores the theoretical link between technological upheavals and the dynamics of production and income distribution. Recent empirical work documents rising skill premia and earnings inequality that coincide with the diffusion of information technologies; see, for example, Levy and Murnane (1992). As Greenwood and Yorukoglu (1997) report, this phenomenon is not without precedent: similar patterns followed earlier technological revolutions. Likewise, slowdowns in productivity growth immediately followed significant technological improvements such as the advent of electrification and the better known 1970s slowdown associated with information technologies.

What are the economic forces that account for such phenomena? An important aspect of the relationship between technological change and wage distribution dynamics is likely to be found in the way a new technology complements skills. In addition to their labor augmenting aspects, new technologies often entail a shift in the structure of relative demands for different types of abilities. On impact, the earnings capabilities of individuals may either be favorably or unfavorably affected with an advancement in technology. As emphasized by Schumpeter (1942), there is an element of destruction in every act of creation. But since human capital acquisition is endogenous and since the relative return to different skills has changed, one might expect a shift in the pattern of skill acquisition across members of the population as well. Such changes in human capital investment are likely to affect the pattern of adjustment to technological change.

In this paper we develop a model that explicitly incorporates these features. Suppose individuals are endowed with a known and observable idiosyncratic ability which becomes productive only after training occurs. Because training is time consuming, only more able individuals seek training. Conversely, lower ability types work as unskilled labor. In a steady state with competitively determined wages, the level of production and the distribution of labor market earnings follow naturally.

Technological change, not surprisingly, disrupts this steady state. To capture its disruptive nature, technological change is considered here to be a fundamental alteration of the way in which workers carry out production. Progress (due to developments such as rail travel, electricity, and the internal combustion engine) involves radically new techniques rather than frequent improvements in existing methods. To highlight this aspect, suppose for now that technological change does not alter the distribution of individual capabilities but randomly reassigns the values of abilities for trained and untrained individuals.[1] Because aggregate capabilities are unaffected, reshuffling does not alter the steady state. It does, however, temporarily move production below the long run level. Output falls as those trained (previously high ability types) now possess overall only average ability and are therefore less productive. Output also falls as those who were untrained but now possess high productive abilities leave unskilled jobs and seek training.

After the initial drop, two effects determine the dynamic path of output. As the new high ability types acquire training and enter high productivity jobs, output rises. Conversely, as trained workers in skilled jobs but with relatively low current capabilities retire from the market, output falls. Individuals of this ability would normally not acquire training. However, they acquired skills before the change occurred and are still capable of skilled work albeit relatively low productivity skilled work. As these trained workers leave the market and are replaced with similar ability types who do not become trained, productivity and output fall.

[1] Eicher (1996) and Heckman, Lochner and Taber (1998) also study the interactions of endogenous human capital, technological change and the distribution of earnings. Heckman et al develop a dynamic general equilibrium model with skilled and unskilled sectors as in this paper and quantitatively explore the implications of technological change. The fundamental difference between their model (as well as Eicher's study) and the approach here is that in this paper technological change involves the shuffling of abilities whereas the Heckman et al specify a smooth, non-disruptive process.

The net effect of these two flows on aggregate output naturally depends on the relative rates of training and of retirement. The effects may offset each other and there may be a monotonic convergence toward the steady state. Alternatively, if training occurs quickly and retirement is relatively slow, output will rise rapidly overshooting its steady state level before eventually returning this level as the deadwood die off. This situation provides a mechanism for generating cyclical patterns.

The reallocation of workers across sectors has further implications for the evolution of employment (sectoral as well as aggregate) and hence productivity and the distribution of income. Again, the rate of redistribution of resources across sectors determines the path of these variables. In some cases, the retirement of obsolete skilled workers complements the effects of training and there is monotone convergence; in others, the two effects counteract and again result in overshooting.

2 The Model

2.1 The Economic Environment

Time is discrete and the horizon is infinite: $t = 0, 1, 2, ..., \infty$. In each period, there is a continuum of individuals with mass normalized to unity. Each person faces a constant probability $0 < \delta < 1$ of death in each period. In the event of death, the individual is replaced with a descendant so that the population remains constant. Individual preferences at any date t are given by:

$$E_t \sum_{j=0}^{\infty} \beta^j (1-\delta)^j c_{t+j} \tag{1}$$

where c_n denotes consumption or income in period n, and $0 < \beta < 1$ is a common discount factor.

In each period, individuals have an indivisible unit of time which is allocated to various activities. As in the Roy (1951) model, there are two sectors in which workers can generate income: a skilled sector (Sector 1) and an unskilled sector (Sector 2). There is also a learning sector (Sector 3) in which individuals undertake skill acquisition activities at the expense of foregone income in the period of training.

Assume that all individuals have at any time the option of working in the unskilled sector. This activity generates an output $w_2 > 0$, a parameter common to all individuals. If individuals have acquired sufficient training in a particular skill, they may instead choose to work in Sector 1. This activity generates output equal to $w_1 a$, where $w_1 > 0$ is a parameter and $a \geq 0$ represents an individual's 'ability' or 'skill.' We interpret a as reflecting the *market value* of an individual's innate ability using the current technology. Let ability be distributed across the population according to a time invariant distribution function $G(x) = \Pr[a \leq x]$, with associated density function $g(x)$.[2] Assume that both $G(x)$ and $g(x)$ are continuous and differentiable.

Individuals who are not trained to exploit their particular skill may find it desirable to pursue a third option: withdraw from the labor market and engage in learning, i.e., enter Sector 3. Assume individuals are aware of their innate ability a; learning involves understanding the way in which a particular ability is applied to production. Learning is time consuming and, at the individual level, uncertain. In particular, assume that conditional on undertaking learning, knowledge is absorbed with probability $0 < \theta < 1$ and that conditional on successful learning, this knowledge may be applied to production one period in the future.[3]

Technological change is specified as an exogenous labor augmenting process that

[2] There are several other interpretations for this distribution. One possibility is that due to difference in innate characteristics, individuals acquire different skills during training. Given the technology, these characteristics are different inputs for production. Alternatively, individuals may all acquire the same set of skills but vary in their ability to apply these skills given the technology.

[3] It is possible to interpret this period of training as a fixed (and relatively short) period of skill acquisition followed by a period of random job search. In this case, θ represents the probability of finding an acceptable job match.

increases the productivity parameters w_1 and w_2. It also disrupts individual production possibilities. Old economic relationships are altered as the economy diverts resources toward adjusting to the new structure of technology. The disruptive aspects of technological change are modelled here as a random 'shuffling' of abilities; i.e., conditional on the arrival of a new technology, an individual draws a new ability parameter a at random from the distribution $G(\cdot)$. Shuffling, however, does not imply that trained workers become untrained. Although they still possess the skill and ability to produce output, the market no longer values their production as before.[4]

The assignment of new values to individual abilities embodies Schumpeter's (1942) notion of creative destruction. Unlike other formulations of this process (e.g. Howitt and Aghion, 1992, 1994), this specification does not involve the creation and destruction of rents. In this sense, the treatment is aligned with that of Jones and Newman (1995) and Helpman and Rangel (1998). Jones and Newman use a search model in which progress undermines accumulated learning-by-doing. Shuffling follows a shock as workers search and realign themselves over potential matches. Helpman and Rangel distinguish between experience which is useful under only the existing technology and life-long knowledge from formal education.

As the focus of this paper is on technology shocks that are 'significant' but 'rare' events, we will refer to them as 'paradigm shifts.' Assume that individuals attach

[4]Under the new technology, productivity is constant over time - output for each ability does not depend upon the number of trained or untrained workers at a given time t. This specification therefore implicitly assumes that the new production technology is linear in labor inputs. This formulation also assumes that all agents immediately adopt the new technology. The focus of this paper is on performance of the economy as workers react to new methods of valuing their inputs rather than the adoption itself. Some Luddite agents will prefer not to adopt as they are better off with the old techniques that highly value their skills. However, it is possible to deny such agents access to the old technology by introducing capital or land in production. When the new technology is introduced, this capital shifts to the more productive new techniques thereby making the old methods inoperable. This is most readily seen when the amount of capital is fixed and indivisible.

zero probability to the event of a paradigm shift. This behavior is approximately optimal if paradigm shifts occur with sufficiently low probability. Conditional on a paradigm shift, however, aggregate dynamics will follow a perfect foresight path.[5]

2.2 Decision-making

The model's simple recursive structure allows us to cast optimal decision-making in terms of the solution to a dynamic program. There are three value functions associated with optimal decision-making: the value of being a trained individual V; the value of being an untrained individual pursuing an education, S; and the value of being an untrained individual working in the unskilled sector, Q.

For a given (w_1, w_2), the value function for a trained individual with ability a must satisfy:

$$V(a) = \max\{w_1 a, w_2\} + \beta(1-\delta)V(a)$$

which implies:

$$V(a) = \max\{w_1 a, w_2\}/[1-\beta(1-\delta)]$$

Note that all trained individuals with ability $a \geq \omega \equiv w_2/w_1$ will choose to work in the skilled sector, while trained individuals with ability $a < \omega$ will instead choose to work in the unskilled sector.

At the start of each period, an untrained individual has the option of either training or choosing to work in the unskilled sector, independent of any previous decisions. With this option in mind, let

$$K(a) \equiv \max\{S(a), Q(a)\}$$

The value function for an untrained individual with ability a who chooses to pursue

[5]During the period of training, individual dynamics will still contain an element of uncertainty; hence, the expectations operator in equation (**??**).

training this period is given by:

$$S(a) = \beta(1-\delta)\left[(1-\theta)K(a) + \theta V(a)\right]$$

For an untrained individual with ability a who decides to work in the unskilled sector this period, the value of doing so can be written as:

$$Q(a) = w_2 + \beta(1-\delta)K(a)$$

It is straightforward to verify that for a given (w_1, w_2) there exists a threshold ability, a_R, partitioning untrained workers. High ability types ($a > a_R$) pursue training while low ability individuals ($a < a_R$) work in the unskilled sector. Moreover, stationarity implies that those who optimally pursue either training or unskilled work this period will find it optimal (given our assumption on expectations) to anticipate making the same decision in all subsequent periods. Given these decisions, the value function for training then simplifies to:

$$S(a) = \xi V(a) = \xi \max\{w_1 a, w_2\}/[1-\beta(1-\delta)] \qquad a \geq a_R$$

where $\xi \equiv [\beta(1-\delta)\theta]/[1-\beta(1-\delta)(1-\theta)] \in [0,1]$. Likewise, it can be established that the value of unskilled work to a low ability type is given by

$$Q = w_2/[1-\beta(1-\delta)] \qquad a < a_R$$

The productivity of skilled labor does not factor into the calculation of well-being for low ability types since w_1 can only be exploited in the event that a technological disturbance alters the value of ability in such a way that makes skilled work worth pursuing. Such an event is perceived to occur with zero probability. Similarly the value of unskilled work is independent of the individuals potential or ability in the skilled sector, a, and this term is dropped.

We can now characterize the reservation ability level a_R. Workers with ability equal to this threshold are indifferent between unskilled work and training: $S(a_R) =$

Q. It then follows that

$$a_R = \omega/\xi \tag{2}$$

The critical value a_R depends on the ratio $\omega = w_2/w_1$ in a natural way. This threshold increases as productivity in the unskilled sector rises, falls as productivity in the skilled sector rises and remains constant if productivity increases proportionally across sectors. Note as well that $\xi < 1$ so that $a_R > \omega$. Therefore, there will generally exist individuals with ability $\omega < a < a_R$ who would choose to work in the skilled sector conditional on being trained, but who would not pursue an education conditional on being untrained. Finally, note that a_R is decreasing in ξ; this latter parameter is an increasing function of β and θ, and a decreasing function of δ. In other words, as individuals become more 'patient' (an increase in β or a decrease in δ), or if learning becomes easier (an increase in θ), they are more willing to take the time investment required for training; i.e., a_R falls.

3 Transition Dynamics and Steady States

Suppose that the economy experiences an (unanticipated) technological advancement that raises productivity in sector $i = 1, 2$ by a factor $\gamma_i \geq 1 : (w_1', w_2') = (\gamma_1 w_1, \gamma_2 w_2)$. In general, these shocks are arbitrary, but two special cases are of particular interest. A *neutral shock* occurs when $\gamma_1 = \gamma_2 = \gamma > 1$ while a *skill-biased shock* involves $\gamma_1 > \gamma_2 \geq 1$.[6] From (**??**), we immediately have the following simple result.

Result 1. *A neutral technology shock leaves a_R unchanged (i.e., does not affect the training decision), while a skill-biased technology shock reduces a_R (i.e., induces more individuals to seek training).*

[6] Goldin and Katz (1996) present historical evidence suggesting that both skill-biased and unskilled-biased ($\gamma_1 < \gamma_2$) changes have occurred. In more recent decades, technological change appears to skill-biased. See, for example, Autor, Katz and Krueger (1998), Bartel and Lichtenberg (1987), Bound and Johnson (1992), and Murphy and Welch (1992).

Let $\lambda_t(a)$ and $\mu_t(a)$ denote the densities of trained and untrained individuals, respectively, with ability a at date t. These densities are related by the restriction that $g(a) = \lambda_t(a) + \mu_t(a)$. During any period, untrained individuals with $a \geq a_R$ become trained at rate θ; hence $\lambda_t(a)+\theta\mu_t(a)$ represents the stock of trained individuals with $a \geq a_R$ at the end of the period. A fraction $(1-\delta)$ of these persons will survive into the next period. On the other hand, trained individuals with $a < a_R$ simply die off at rate δ. Hence, the evolution of trained workers with ability a is given by:

$$\lambda_{t+1}(a) = \begin{cases} (1-\delta)[(1-\theta)\lambda_t(a) + \theta g(a)] & \text{for } a \geq a_R \\ (1-\delta)\lambda_t(a) & \text{for } a < a_R \end{cases}$$

where use has been made of $g(a) = \lambda_t(a) + \mu_t(a)$. Note that due to the model's structure, individual decision-making (a_R) does not depend on the distribution $\lambda_t(a)$.

Consider some initial distribution $\lambda_0(a)$. For $a \geq a_R$, repeated substitution yields

$$\lambda_t(a) = \phi^t \lambda_0(a) + [\phi^{t-1} + \phi^{t-2} + ... + 1](1-\delta)\theta g(a).$$

where $\phi \equiv (1-\delta)(1-\theta)$. Notice that the term in the square brackets equals $(1 - \phi^t)/(1-\phi)$, so that we can write the evolution of this distribution function as:

$$\lambda_t(a) = \begin{cases} \rho(1-\phi^t)g(a) + \phi^t \lambda_0(a) & \text{for } a \geq a_R \\ (1-\delta)^t \lambda_0(a) & \text{for } a < a_R \end{cases} \tag{3}$$

where $0 < \rho \equiv (1-\delta)\theta/(\delta+\theta-\delta\theta) < 1$. Equation (**??**) implies that as $t \to \infty$, $\lambda_t \to \lambda$, where λ is given by:

$$\lambda(a) = \begin{cases} \rho g(a) & \text{for } a \geq a_R \\ 0 & \text{for } a < a_R \end{cases} \tag{4}$$

Likewise, the steady state distribution of untrained individuals, μ, is given by $\mu(a) = g(a) - \lambda(a)$.

The distributions λ and μ can be used to determine long run employment and output in each sector. In particular, conditional on a_R, the steady state measure of trained individuals working in the skilled sector is given by:

$$N_1 = \int_{a_R}^{\infty} \lambda(a)da = \rho\,[1 - G(a_R)] \tag{5}$$

so that steady state output in this sector is

$$Y_1 = w_1 \int_{a_R}^{\infty} a\lambda(a)da = w_1 N_1 E\left[a \mid a \geq a_R\right] \tag{6}$$

In the unskilled sector, the steady state measure of working individuals:

$$N_2 = \int_0^{a_R} \mu(a)da = G(a_R). \tag{7}$$

generate unskilled output:

$$Y_2 = w_2 G(a_R) = w_2 N_2 \tag{8}$$

Note that N_1 is decreasing in a_R while N_2 in increasing in a_R. Further it is straightforward to demonstrate that total employment $N \equiv N_1 + N_2$ is increasing in a_R.

The model can also be used to evaluate the effect of technological change on productivity, the skill premium and the distribution of earnings. As all workers have the same ability for unskilled work, steady state productivity in this sector is simply: $P_2 = Y_2/N_2 = w_2$. On the other hand, due to self-selection, steady state productivity in the skilled sector depends on not only w_1 but also on the average ability of those who enter Sector 1: i.e. $P_1 = Y_1/N_1 = w_1 E[a \mid a > a_R]$. Given that workers earn their marginal products, the ratio of skilled to unskilled productivity provides a measure of the long run wage differential or skill premium:

$$\Pi = \frac{P_1}{P_2} = \frac{w_1 E[a \mid a \geq a_R]}{w_2} = \frac{\int\limits_{a_R}^{\infty} ag(a)da}{\omega[1 - G(a_R)]} \tag{9}$$

Finally, we examine the steady state distribution of earnings (among those who are employed). Let $H(y)$ denote the fraction of employed individuals with earnings less than or equal to y; this distribution is given by:

$$H(y) = \begin{cases} 0 & 0 \leq y < w_2 \\ G(a_R)/N & w_2 \leq y < a_R w_1 \\ [(1-\rho)G(a_R) + \rho G(y/w_1)]/N & a_R w_1 \leq y < \infty. \end{cases} \tag{10}$$

3.1 Comparing Steady States

Following a technological shock, changes in the steady state decompose into two effects: (i) those that arise when the threshold ability is held constant and (ii) those due to shifts in a_R. Since a_R determines the allocation of workers to sectors, the first effect captures pure efficiency improvements associated with a technological shock while the second captures the reactions of individuals as they self-select across sectors. Moreover, Result 1 implies that a neutral technology shock does not generate a long run reallocation of workers across sectors so that such a paradigm shift yields only pure efficiency effects. These are easy to discern and stated without proof:

Result 2 *Following a neutral technological improvement, the new steady state is characterized by: (i) unchanged sectoral employment levels; (ii) increased sectoral and aggregate output; (iii) increased sectoral productivity levels; (iv) an unchanged skill premium; and (v) a new wage distribution that stochastically dominates the initial distribution.*

The consequences of self-selection which accompany non-neutral breakthroughs are more involved. A new technology that complements skill leads individuals with marginal ability levels to now undertake training. Not surprisingly, long run skilled employment rises, while long run unskilled employment falls. Perhaps a bit more surprising is the fact that aggregate employment falls. It appears as if new technology reduces the demand for labor. Of course, what is in fact happening is that more time is being allocated to the training sector.

These shifts in employment alter output, productivity and the distribution of earnings in a manner beyond that arising from a neutral shock. Steady state output in the skilled sector increases not only as the existing workforce becomes more productive but also as employment in the skilled sector increases. In the unskilled sector, productivity gains offset to some degree the loss from out-migration. The net effect depends the magnitude of the productivity gains. Migration dominates small gains in productivity and hence sectoral output declines. Large productivity rises, however, lead to increased sectoral output. Although at this level of generality it appears as if aggregate output may either rise or fall, the combined effects leads to increased aggregate output.[7]

Migration also generates ambiguous productivity effects in the skilled sector. The shock $\gamma_1 > 1$ not only increases output per skilled worker, it also lowers a_R and hence the average quality of workers in the skilled sector. Whether productivity as well as the skill premium rises or falls depends on the relative strength of these two effects. If the number of new skilled workers (those who would not have entered before the shock) is large relative to those already in training, then productivity and the skill premium will fall.[8] Nonetheless, the new low-ability skilled workers are more productive than they were as unskilled workers so aggregate productivity rises. Ambiguity, however, is not observed in the wage distribution as H 'spreads to the right' following a technological upheaval. The new distribution stochastically dominates the old. Observe, however, that a skill-biased shock yields a qualitatively

[7] Adding the two sectors and differentiating produces

$$\frac{d(Y_1+Y_2)}{d\gamma_1} = w_1\rho \int_{a_R}^{\infty} ag(a)da - \frac{(1-\beta)\gamma_2 w_2 a_R g(a_R)}{\beta\gamma_1^2(\delta+\theta-\delta\theta)} > 0$$

where use has been made of a_R from (**??**). Likewise,

$$\frac{d(Y_1+Y_2)}{d\gamma_2} = \frac{(1-\beta)w_2 a_R g(a_R)}{\beta(\delta+\theta-\delta\theta)} + w_2 G(a_R) > 0$$

[8] Differentiating skilled productivity with respect to γ_1 and accounting for $da_R/d\gamma_1$ yields

$$\frac{\partial(Y_1/N_1)}{\partial\gamma_1} = w_1\left\{E(a \mid a > a_R') - \frac{a_R' g(a_R')}{1-G(a_R')}\left[E(a \mid a > a_R') - a_R'\right]\right\}$$

where a_R' is the reservation ability associated with the pair $(\gamma_1 w_1, \gamma_2 w_2)$. Depending on the magnitude of $g(a_R')/[1-G(a_R')]$, skilled productivity may either rise or fall. Alternatively, for a sufficiently low a_R (and hence high w_1), this derivative is positive. Since unskilled productivity does not decline, the skill premium Π declines if skilled productivity either falls or rises below some limit.

different shape for the new distribution than a neutral shock. From this discussion, the key long run results are summarized as:[9]

Result 3 *Following a skill-biased technological improvement, the new steady state is characterized by: (i) an increase in skilled employment and training activities, but a decrease in unskilled and aggregate employment; (ii) an increase in skilled output (iii) a decrease in unskilled output if and only if γ_2 is sufficiently close to unity (iii) a skill premium that is higher or lower than before; and (iv) a new wage distribution that stochastically dominates the initial distribution.*

3.2 Transition Dynamics

Imagine beginning the economy in a steady state for a given (w_1, w_2). Suppose next that in some period, which we label arbitrarily as $t = 0$, the economy experiences a technology shock. To highlight the importance of shuffling on the adjustments across sectors (the direct effects from the shock), assume that the technological advancement is neutral. In this case, $\gamma_1 = \gamma_2 = \gamma > 1$, a_R remains constant, and there is no long run re-allocation of workers across sectors.

Just prior to the shock, the distribution of trained individuals is given by the steady state condition for λ defined in (**??**). In the impact period, the measure N_1 of trained individuals find the value of their abilities rearranged randomly according to G; hence, the $t = 0$ distribution of trained individuals following the shock is:

$$\lambda_0(a) = N_1 g(a) \quad \text{for all } a.$$

Inserting $\lambda_0(a)$ into equation (**??**), it then follows that for $t \geq 0$:

$$\lambda_t(a) = \begin{cases} \rho[1 - \phi^t G(a_R)]g(a) & \text{for } a \geq a_R \\ (1-\delta)^t N_1 g(a) & \text{for } a < a_R \end{cases}$$

[9]These findings follow from straightforward manipulation of equations (**??**)-(**??**), further details of which can be found in Andolfatto and Smith (1999).

The speed of convergence for this distribution depends on parameters θ and δ which measure, respectively, the ease of learning and the rate at which individuals exit (and enter) the economy.[10] In particular, if training occurs rapidly while individuals retire slowly (θ close to one and δ near zero), then early on the number of trained individuals will exceed the long run level. The number of trained individuals with abilities above a_R moves rapidly toward the steady state level. Coexisting with these people will be the old guard of trained workers with abilities that were shuffled below the threshold level. Among these trained individuals, those who remain in the skilled sector $\omega < a < a_R$ have higher output than their direct descendents who eventually replace them. We now turn to the implications of this process.

Sectoral and aggregate employment follow from appropriately summing over $\lambda_t(a)$ and $\mu_t(a)$. In particular, let N_{jt} denote the number of individuals active at date t in sector $j = 1, 2, 3$, where Sector 3 is training. Sector 1 skilled employment is given by:

$$N_{1t} = \int_{\omega}^{\infty} \lambda_t(a)da = N_1[1 - \phi^t G(a_R)] + (1-\delta)^t N_1 [G(a_R) - G(\omega)] \tag{11}$$

while Sector 2 unskilled employment is given by:

$$N_{2t} = \int_0^{\omega} \lambda_t(a)da + \int_{\omega}^{a_R} [g(a) - \lambda_t(a)]\, da = G(a_R) - (1-\delta)^t N_1[G(a_R) - G(\omega)] \tag{12}$$

The total working population at t is simply $N_t = N_{1t} + N_{2t}$ while the number of individuals undertaking skill acquisition activities is the residual $N_{3t} = 1 - N_t$.

Output in each sector evolves in a similar manner. For $t \geq 0$, output and hence total income in the skilled sector evolve according to:

$$\begin{aligned} Y_{1t} &= \gamma w_1 \int_{\omega}^{\infty} a\lambda_t(a)da = \gamma w_1 \int_{\omega}^{a_R} a\lambda_t(a)da + \gamma w_1 \int_{a_R}^{\infty} a\lambda_t(a)da \\ &= \gamma w_1 (1-\delta)^t N_1 \int_{\omega}^{a_R} ag(a)da + \gamma w_1 \rho \left[1 - \phi^t G(a_R)\right] \int_{a_R}^{\infty} ag(a)da. \end{aligned} \tag{13}$$

The first term represents output from the 'deadwood,' i.e. those individuals who continue to work in the skilled sector following a shuffle shock only by virtue of the

[10] Recall that $\phi \equiv (1-\theta)(1-\delta)$ and $\rho \equiv (1-\delta)\theta/(\delta + \theta - \delta\theta)$.

fact that they find themselves already trained. Given their new ability, these workers would not chose to become trained; however, they already are. Moreover, their ability is such that they find it worthwhile to continue in the skilled sector.

Output in the unskilled sector is similarly calculated:

$$\begin{aligned} Y_{2t} &= \gamma w_2 \int_0^{\omega} \lambda_t(a)da + \gamma w_2 \int_0^{a_R} [g(a) - \lambda_t(a)]da \\ &= \gamma w_2 (1-\delta)^t N_1 G(\omega) + \gamma w_2 G(a_R) \left[1 - (1-\delta)^t N_1\right]. \end{aligned} \tag{14}$$

Here too, the first term represents a type of deadwood, but now these are the previously high-ability and trained individuals who relocate to the unskilled sector. The second term represents the output of unskilled workers who never acquired training. Given the behavior of output and employment, sectoral and aggregate productivity follow in the natural way: $P_{jt} = Y_{jt}/N_{jt},\ j = 1, 2$ and $P_t = Y_t/N_t$, respectively.

The skill premium, out of steady state, has two potential interpretations: (i) average earnings among workers in skilled jobs relative to the unskilled wage and (ii) the expected wage of all trained workers relative to the wage of untrained workers. In the long run, these measures both equal Π but in an evolving economy, some trained individuals are found working in the unskilled sector. In particular, after a technology shock, the value of some trained workers ability is less than ω. In this case, these workers switch into the unskilled sector causing the two measures of the skill premium to differ. Since unskilled wages are constant, the evolution of the premium defined by skilled wages will exactly mirror that of skilled productivity. Therefore, the alternative definition - the ratio of wages for trained workers to those of untrained (and employed) workers - is used here:

$$\Pi_t = \frac{\gamma w_2 \int\limits_0^{\omega} \lambda_t(a)da + \gamma w_1 \int\limits_{\omega}^{\infty} a\lambda_t(a)da}{\int\limits_0^{\infty} \lambda_t(a)da} \tag{15}$$

From these equations of motion, it is now possible to determine both the immediate impact of the technological change and the ensuing adjustments to the steady state. Consider first the impact effect of the technology shock. When the shock occurs, some individuals suddenly experience an increase in the demand for their particular skill, while others find their skills to be less valuable. In response, some workers immediately relocate. From equations (**??**) and (**??**), skilled and unskilled employment in the period just following the shock are given by:

$$N_{1\,0} = [1 - G(\omega)]\, N_1 < N_{1.}$$

$$N_{2\,0} = (1 - N_1)G(a_R) + N_1 G(\omega) < G(a_R) = N_{2.}$$

Relocating workers cause sectoral and hence aggregate employment to contract in the impact period. The skilled sector now contains trained individuals with abilities above the reservation threshold as well as deadwood. The probability that a trained worker becomes Sector 1 deadwood in the period of the technology shock is given by $G(a_R) - G(\omega)$, so that $[G(a_R) - G(\omega)]N_1$ represents the mass of deadwood in Sector 1 in the impact period of the shock. Other trained workers experience a more drastic wage decline that generates a transition into unskilled work; this initial flow is given by $G(\omega)N_1$. (No skilled workers flow into Sector 3 since they are already trained.) As there is no corresponding inflow of workers into the skilled sector in the impact period, skilled employment declines.

As for untrained workers, some find that the value of their ability has increased to such an extent that training is now desirable; the flow of these workers into training is given by $[1 - G(a_R)]\, N_2$. There is also a flow of individuals from the training sector into unskilled work: the new technology makes some workers currently in training abandon the effort. This flow is given by $(1 - N_1 - N_2)G(a_R)$. As worker outflows exceed worker inflows, unskilled employment declines on impact.

The impact effect of a technology shock on output, productivity and the skill premium depends on the size of the productivity improvement.[11] For γ sufficiently

[11] The impact effect can also be expected to depend on the speed with which new technology is adopted; see Andolfatto and MacDonald (1998).

close to one, the decline in aggregate employment outweighs productivity gains so that aggregate (and sectoral) output falls in absolute terms. While unskilled productivity rises by the factor γ, the immediate effect on output per skilled worker is ambiguous. The positive γ effect is counteracted by a dilution in the average quality by deadwood. In any case, the net effect on the skill premium is negative. These findings are summarized as follows. (Derivations are found in the appendix.)

Result 4 *In the impact period of a neutral technological advancement: (i) sectoral and aggregate employment declines; (ii) sectoral and aggregate output declines relative to trend (and declines absolutely for γ sufficiently close to unity); (iii) productivity in the unskilled sector rises, while productivity in the skilled sector falls relative to trend (and falls absolutely for γ sufficiently close to unity); (iv) the skill premium declines.*

The adjustment path toward this new steady state is perhaps of more fundamental interest than the immediate changes. The transition properties are summarized as follows. (Derivations are outlined in the appendix.)

Result 5 *Following the impact period of a neutral technological advancement: (i) unskilled output and employment converge monotonically to the new steady state; (ii) skilled employment and output overshoot their long run levels; and (iii) skilled and aggregate productivity as well as the skill premium increase monotonically over time.*

Productivity in the unskilled sector remains constant due to the assumption of homogeneity of unskilled work.[12] For unskilled employment, over time workers flow into this sector as the descendants of deadwood ($\omega < a \leq a_R$) from the skilled sector

[12] Heckman and Honore (1990) provide a more detailed description for heterogeneity in Roy's model but without training and technological shocks, the focus here.

arrive. Since there are no further outflows into training after $t = 0$, the rise in the total number of workers increases output in the unskilled sector.

In the skilled sector, the process is more complicated. New workers enter as they become trained. Given the large inflow into training from the unskilled sector at $t = 0$, the flow into the skilled sector in subsequent periods is above the long term rate. Employment and output therefore tend to rise. At the same time, deadwood die and are replaced by descendents who take up unskilled work. This outflow which drives the revival in unskilled output counteracts the rise in the inflow from the training sector. The net effect varies over time so that overshooting of output in the skilled sector occurs. Employment in the skilled sector likewise does not move monotonically to the new steady state. As shown in the proof, employment in the skilled sector peaks before output.

As demonstrated in Figure 1, aggregate output, $Y_t = Y_{1t} + Y_{2t}$, can follow a similar pattern although it will be dampened by the monotonic rise in unskilled output.[13] Aggregate overshooting arises when the inflow from training adjusts quickly while the outflow of deadwood is relatively slow: a high θ and low δ as described above. On the other hand, the net effects from unskilled and skilled production may balance and there can be monotonic convergence to the new trend. Moreover, since deadwood switch sectors without delay while more individuals become trained, total employment necessarily rises after the initial drop. As a result, although skilled employment peaks before the peak in production, this relationship does not hold between aggregate output and aggregate employment.

As relatively less productive or below average deadwood die off, above average trained workers enter the skilled sector; therefore, skilled productivity monotonically rises over time albeit to a potentially lower steady state level. These flows similarly

[13] In this example, the technology shock is the pure shuffle case ($\gamma = 1$) so the steady state levels are unaffected by the change. Output is measured relative to this steady state level. Parameter values are $\beta = .96$, $\delta = .1$, $\theta = .35$, $w_1 = 2$, $w_2 = 1$, and $G(a) = 1 - \exp(-a)$.

cause average wages of trained workers to rise. As the deadwood in the unskilled sector retire, they are replaced by descendants who do not seek training. Together these three flows all contribute toward increasing the skill premium.

The transition path of the wage distribution reflects for the most part the evolution of the skill premium. Suppose for convenience a pure shuffle shock occurs ($\gamma_1 = \gamma_2 = 1$) so that the steady income distribution as well as a_R are unchanged.[14] Summing over the earnings of all individuals by ability, the distribution of individuals with earnings less than or equal to y can be written as:

$$H_t(y) = \begin{cases} 0 & 0 \leq y < w_2 \\ \{G(a_R) + (1-\delta)^t N_1[G(\frac{y}{w_1}) - G(a_R)]\}/N_t & w_2 \leq y < a_R w_1 \\ \{G(a_R) + \rho[1-(1-\delta-\theta+\delta\theta)^t G(a_R)][G(\frac{y}{w_1}) - G(a_R)]\}/N_t & a_R w_1 \leq y < \infty. \end{cases} \tag{16}$$

Result 6 *Consider a pure shuffle shock* ($\gamma_1 = \gamma_2 = 1$)*. (i) For* $y \geq a_R w_1$, $H_t > H_{t+1}$ *for all* t*. (ii) For all* $y < a_R w_1$, $H_t < H_{t+1}$ *for sufficiently large* t*, while* $H_0 - H$ *is ambiguous and depends on* y*.*

(See appendix.) As seen in Figure 2, Result 6 implies that the high end of the earnings distribution becomes 'flatter' over time as well paid, trained workers become more plentiful. For the low end consisting of the deadwood and unskilled (workers with $y < a_R w_1$), adjustment is likely to be uneven. It depends on several factors including date and income level. The proportions of deadwood and untrained may initially fall as training raises the working population; later these proportions will rise. At later dates, such changes give the impression of more earnings inequality although for a given and fixed ability level, workers are better off following a shock.[15]

[14]For this special case, the impact effect can be found by comparing the $t = 0$ distributions with the distributions as $t \to \infty$. Note as well that due to the long run shifts in the earnings distributions, characterizing the adjustment patterns for the earnings distribution given $\gamma > 1$ requires even in this simple model considerable and tedious elaboration.

[15]The welfare properties are straightforward to determine from the value functions. Unskilled

4 Discussion

When a technological advancement appears on the scene, it is not absorbed smoothly by the economy. It disrupts aggregate production possibilities and differentially alters the productivity of individuals. Moreover, adjusting to these changes is a time consuming and costly process. As Schumpeter (1927) notes, a technological breakthrough "must fundamentally alter the bases of calculation and upset the existing equilibrium beyond the possibility of all people adapting themselves successfully by marginal variations." This paper presents a theoretical framework explicitly incorporating these factors and provides a mechanism for understanding the way in which this upheaval affects economic performance over time.

In the long run, changes in relative wages across sectors determines employment re-allocations hence sectoral output and productivity. Workers shifting sectors are below the average quality of those already employed in the new sector and they therefore mask long run improvements in productivity and undermine the measurement of the skill premium. In the very short run, the shuffling effects associated with a breakthrough diminish the immediate impact of the shock. Employment falls as large numbers of unskilled workers pursue training. Coupled with the loss of the self-selection benefits in the skilled sector, this reduction counteracts the improvements in technology leading to ambiguous effects, especially with respect to output.

Between the very short and the very long term, a *replacement effect* occurs as trained workers with relatively low skills are replaced by workers who do not become trained, do not enter the skilled sector and therefore produce and earn less. On the other hand, a *training effect* also occurs as workers with newly valued abilities switch

workers will always welcome the upside of this upheaval. Trained workers and those in training whose abilities become drastically less valuable will find the technological shock harmful. Those trained workers whose skills gain value and those who do not lose out too much will benefit. See Andolfatto and Smith (1999) for a complete characterization of Luddites and technophobes.

from the unskilled to the skilled sector. As this change occurs, output and earnings rise. The net effect varies over time leading to overshooting of the new steady state in the skilled sector which in turn may lead to aggregate overshooting.

These qualitative properties link changes in productivity, the skill premium, wage dispersion, sectoral employment and output growth with dramatic breakthroughs in technology and it is possible to search for these connections in the data.[16] For example, as Krusell et al (1997) point out, both the quantity and relative wages of skilled workers have grown significantly since 1980 while Eicher (1996) highlights the fluctuating pattern of relative wages and quantity of skilled labor. Both observations are consistent with the effects of a skill-biased technological change in this paper. The model also predicts concurrent changes in the earnings distribution, output, productivity and sectoral employment. In this way, the model connects the existence and timing of shifts in observed economic outcomes. However, any such empirical work requires consideration of the appropriate time dimension. Given the profound nature technological shock, overshooting is unlikely to correspond to regular business cycles but rather to changes Schumpeter (1927) labels "long wave."

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[16]Empirical implications can also be drawn from a more general view of the unskilled. The obvious view and the one pitched so far is that unskilled work is 'brawn' and skilled labor 'brain' production. One could alternatively interpret unskilled production as a sector unaffected by technological upheavals. The resulting worker flows then become more general sectoral flows.

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6 Appendix

6.1 Proof of Result 4

(i) Follows directly from the discussion in the text.

(ii) From (**??**) and (**??**) , comparing skilled and unskilled output on impact relative to trend gives

$$\begin{aligned} Y_{1\,0} - \lim_{t\to\infty} Y_{1t} &= \gamma w_1 N_1 \int_{\omega}^{a_R} ag(a)da + \gamma w_1 \rho[1 - G(a_R)] \int_{a_R}^{\infty} ag(a)da - \gamma w_1 \rho \int_{a_R}^{\infty} ag(a)da \\ &= \gamma w_1 N_1 \left[\int_{\omega}^{\infty} ag(a)da - E(a \mid a > a_R) \right] < 0 \end{aligned}$$

and

$$Y_{2\,0} - \lim_{t\to\infty} Y_{2t} = \gamma w_2 N_1 [G(\omega) - G(a_R)] < 0$$

The absolute impact on skilled output follows from comparing Y_1 from (**??**) given w_1 and w_2 with $Y_{1\,0}$:

$$\begin{aligned} Y_{1\,0} - Y_1 &= w_1 \rho (1 - G(a_R)) \left[\gamma \int_{\omega}^{\infty} ag(a)da - \int_{a_R}^{\infty} \frac{ag(a)}{1 - G(a_R)} da \right] \\ &= w_1 N_1 \{ \gamma (1 - G(\omega)) E[a \mid a \geq \omega] - E[a \mid a \geq a_R] \} \end{aligned}$$

$$Y_{1\,0} - Y_1 > 0 \Leftrightarrow \gamma > \frac{E[a \mid a \geq a_R]}{(1 - G(\omega))E[a \mid a \geq \omega)]} > 1$$

Likewise, from (**??**), the absolute change in unskilled output is:

$$Y_{2\,0} - Y_2 = w_2 [\gamma N_1 G(\omega) + \gamma (1 - N_1) G(a_R) - G(a_R)]$$

$$Y_{2\,0} > Y_2 \Leftrightarrow \gamma > \frac{G(a_R)}{(1 - N_1) G(a_R) + G(\omega) N_1} > 1$$

(iii) For output per skilled worker

$$P_{1\,0} - \lim_{t\to\infty} P_{1t} = \gamma w_1 \left[E(a \mid a > \omega) - E(a \mid a \geq a_R) \right] < 0$$

$$\begin{aligned} P_{1\,0} - P_1 &= w_1 \left[\gamma E(a \mid a > \omega) - E(a \mid a \geq a_R) \right] \\ P_{1\,0} &> P_1 \Leftrightarrow \gamma_1 > \frac{E(a \mid a \geq a_R)}{E(a \mid a > \omega)} > 1 \end{aligned}$$

On the other hand, $P_{2\,0} - P_2 = \gamma_2 w_2 - w_2 < 0$.

(iv) Follows from (iii)

6.2 Proof of Result 5

(i) From (??) and (??), as the time interval becomes infinitesimally small, unskilled employment and output become:

$$N_{2t} = N_2 - e^{-\delta t} N_1 [G(a_R) - G(\omega)]$$

$$Y_{2t} = \gamma_2 w_2 \{N_1 G(\omega) e^{-\delta t} + G(a_R)[1 - N_1 e^{-\delta t}]\}$$

which are increasing over time.

(ii) Similarly, from (??) and (??) skilled employment and output are

$$N_{1t} = N_1 \left\{ [1 - G(a_R) e^{-(\delta+\theta)t}] + e^{-\delta t} [G(a_R) - G(\omega)] \right\}$$

$$Y_{1t} = \gamma_1 w_1 \left\{ e^{-\delta t} N_1 \int_{\omega}^{a_R} a g(a) da + \frac{\theta}{\delta + \theta} [1 - G(a_R) e^{-(\delta+\theta)t}] \int_{a_R}^{\infty} a g(a) da \right\}$$

and hence:

$$\dot{N}_{1t} = N_1 e^{-\delta t} \left\{ (\delta + \theta) G(a_R) e^{-\theta t} - \delta [G(a_R) - G(\omega)] \right\}$$

$$\dot{Y}_{1t} = \gamma_1 w_1 e^{-\delta t} \left\{ -\delta N_1 \int_{\omega}^{a_R} a g(a) da + \theta N_2 e^{-\theta t} \int_{a_R}^{\infty} a g(a) da \right\}$$

Note that $\dot{N}_{1\,0} > 0$ while N_t peaks at $\dot{N}_{1t^{**}} = 0$ where

$$t^{**} \equiv -\frac{1}{\theta} \ln \left[\frac{\delta}{\delta + \theta} \frac{G(a_R) - G(\omega)}{G(a_R)} \right] > 0$$

Since $N_1 = \frac{\theta}{\delta+\theta}[1 - G(a_R)]$ and $N_2 = G(a_R)$

$$\dot{Y}_{1\,0} > 0 \Leftrightarrow \frac{\delta + \theta}{\delta} = 1 + \frac{\theta}{\delta} > \frac{E(a \mid a < a_R)}{E(a \mid a > a_R)} - \frac{\int_0^{\omega} a g(a) da}{G(a_R) E(a \mid a > a_R)}$$

As $E(a \mid a < a_R)/E(a \mid a > a_R) < 1$, the inequality holds. Similarly, skilled output peaks at t^* defined by

$$t^* \equiv \frac{-1}{\theta} \ln \left\{ \frac{\delta}{\delta + \theta} \frac{1 - G(a_R)}{G(a_R)} \frac{\int_{\omega}^{a_R} a g(a) da}{\int_{a_R}^{\infty} a g(a) da} \right\} > 0$$

(iii) At time t, productivity in the skilled sector can be expressed as

$$\frac{Y_{1t}}{N_{1t}} = \gamma w_1 \frac{N_1 e^{-\delta t} \int_{\omega}^{a_R} ag(a)da + \frac{\theta}{\delta+\theta}\left[1 - e^{-(\delta+\theta)t} G(a_R)\right] \int_{a_R}^{\infty} ag(a)da}{N_1 \left\{[1 - G(a_R)e^{-(\delta+\theta)t}] + e^{-\delta t}[G(a_R) - G(\omega)]\right\}}$$

The derivative with respect to time is

$$\begin{aligned} \dot{P}_{1t} &= \frac{\gamma w_1 N_1}{N_{1t}{}^2} \times \Big\{ N_{1t} \cdot \Big(-\delta I_1 e^{-\delta t} + \theta e^{-(\delta+\theta)t} G(a_R) I_2 / N_1 \Big) \\ & \quad - \Big((\delta+\theta) e^{-(\delta+\theta)t} G(a_R) - \delta e^{-\delta t}[G(a_R) - G(\omega)] \Big) \cdot Y_{1t} \Big\} \end{aligned}$$

where $I_1 = \int_{\omega}^{a_R} ag(a)da$ and $I_2 = \int_{a_R}^{\infty} ag(a)da$. After some algebra, this equation yields

$$\frac{N_{1t}{}^2 \cdot \dot{P}_{1t}}{\gamma w_1 N_1{}^2 \left[G(a_R) - G(\omega)\right] \left(\delta + \theta N_2 e^{-(\delta+\theta)t}\right)} = E(a \mid a > a_R) - E(a \mid a_R > a > \omega) > 0$$

As unskilled productivity is constant over time, aggregate productivity follows a pattern similar to that of productivity in the skilled sector.

Consider the skill premium. From (**??**) differentiation and manipulation give

$$\dot{\Pi}_t = \frac{N_1 \theta}{\Upsilon^2 (\delta + \theta)} \left[\delta e^{-\delta t} + \theta N_2 e^{-\theta t}\right] \times$$

$$\left\{ \gamma_1 w_1 E(a \mid a > a_R) - \gamma_2 w_2 \int_0^{\omega} g(a)da - \gamma_1 w_1 \int_{\omega}^{a_R} ag(a)da \right\} > 0$$

where $\Upsilon = \int_0^{\infty} \lambda_t(a)da$

6.3 Proof of Result 6

From (**??**)

$$\dot{H}_t = [N_t \dot{D}_t - \dot{N}_t D_t]/N_t{}^2$$

where in the limit as the time interval becomes infinitesimally small, $D_t(y)$ is the measure of employed individuals with income less than y: $D_t(y) \equiv H_t(y)/N_t$. Two cases arise from this specification.

(i) For $y > a_R w_1$,

$$\dot{H}_t = \frac{-\theta e^{-(\delta+\theta)t} G(a_R)^2 [1 - G(y/w_1)]}{{N_t}^2} < 0$$

Note as well that $\partial \dot{H}_t / \partial y > 0$.

(ii) For $y < a_R w_1$

$$\begin{aligned}\dot{H}_t \quad = \quad & \frac{-N_{1t} e^{-\delta t}}{{N_t}^2} \{\delta[G(a_R) - G(y/w_1)] G(a_R) \\ & + \delta[G(a_R) - G(y/w_1)] N_1 [1 - e^{-(\delta+\theta)t} G(a_R)] \\ & - G(a_R)^2 (\delta + \theta) e^{-\theta t} + (\delta + \theta) N_1 G(a_R) [G(a_R) - G(y/w_1)] e^{-(\delta+\theta)t} \Big\}\end{aligned}$$

so that $\dot{H}_t < 0$ as $y \to a_R w_1$. However, for sufficiently large t, $\dot{H}_t < 0$ for all $y < a_R w_1$. For $t = 0$, the sign of $\dot{H}_t$ equals that of

$$-G(a_R) \left[\delta G(y/w_1) + \theta G(a_R)\right] \left[(G(a_R) - G(y/w_1)) N_1 (\delta + \theta G(a_R))\right]$$

This term becomes negative as $y \to a_R w_1$. However for $y = w_2$, this term may be positive for some parameter values.