Reduced Bit-Rate Uniform Quantization for SPIHT Encoding

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Abstract

SPIHT is a scalar quantizer for images that have undergone subband decomposition by wavelet transform. In this letter, reduced bit-rates have been achieved by merging the first and second passes of the already efficient SPIHT algorithm. The modification requires neither entropy encoding nor significant increase to the algorithmic complexity.

1 Introduction

Set Partitioning in Hierarchical Trees (SPIHT) [1] is an established method of uniformly quantizing the coefficients of a wavelet-transformed image. SPIHT progressively transmits bit-planes in successive passes, included an embedded bit-map of significant coefficient sets with each pass. SPIHT compression is achieved [2] by coding sets that share a common characteristic with a single bit in the bit-map. The choice of aggregate set is made via quad-tree structures according to the structure of input images and the transform itself. This letter shows that for many natural images a modest bit-rate improvement can be achieved by including a per-coefficient bit when there are few coefficients that are significant (likely to be set) in the first pass.

It should be noted that improvements to the basic SPIHT algorithm are accepted [3] to be difficult (1-2% compression gain in [3]) without increasing the algorithmic or computational complexity, and, therefore, the modification reported in this paper is worthy of use for standard natural images (Markov order-one fields). Modifications reported by others require arithmetic coding, or trellis quantization [4] to gain their effect, with commensurate increases in complexity, making implementation in hardware difficult.

The essence of the improvement is that the first and second pass of the algorithm are merged, thus discarding an embedded significance mapping in the first pass of the algorithm. A simple preliminary test establishes whether the modification to SPIHT is likely to save bits. Eight-out-of-thirteen standard test images[1] were found to be suitable with an improvement of the order of 1-4% in (lossy) compression for higher resolution encoding. The changes to the original algorithm are not incompatible with improvements introduced by others.

2 Description of modification

A 2-D biorthogonal wavelet transform is applied to an image to produce a coarse-to-fine pyramidical structure of sub-images. In the SPIHT scalar quantization algorithm, commencing at the coarsest resolution, the coefficients are tested against a threshold. Compression is achieved by assigning a single bit to any set for which all members are below the threshold (insignificant). Judicious selection of set members through quadtrees that conform to likely transform structures makes insignificant sets more likely. For formal algorithmic details refer to [1].

The threshold is found [3] as

$$T_p = \frac{c_{max}}{2^{p+1}}, \tag{1}$$

where $p = 0, 1, \ldots$, with p being the pass index, and

$$c_{max} = 2^{\lceil \log_2 \max |c_{i,j}| \rceil}, \tag{2}$$

[1]Images available from http://www.sys.uea.ac.uk/Research/researchareas/imagevision/images_ftp/ (last accessed 3.3.03)

where $c_{i,j}$ is the coefficient at coordinate (i, j) in the image. For any coefficient, a significance test is made such that:

$$S_p(c_{i,j}) = \begin{cases} 1, & \max |c_{i,j}| \geq T_p \\ 0, & \text{otherwise} \end{cases}$$

In effect, the threshold is half the largest integer power of two that exceeds the maximum coefficient grey-scale value.

Unfortunately, in some images there are few coefficients in the initial pass that exceed this threshold. If just one coefficient in a set were to exceed the threshold in any set, it would still necessitate transmission of the complete significance bit-map for that set in order to identify that coefficient.

To determine whether that is likely, the simple test $c_{max} < \frac{3T_0}{2}$ is introduced. If the test is satisfied, then a reduced threshold for pass zero is assumed, namely:

$$T_0' = \frac{c_{max}}{2^2}. \tag{3}$$

For those coefficients that become newly significant in the initial pass, a bit is used to indicate whether those coefficients would also exceed the threshold of T_0 from (1), which would previously have been applied. The extra bits transmitted are traded-off against not transmitting some bit-maps, as a consequence of effectively merging two passes. In all images compressed by the modified scheme (Section 3), reduction in bit-rate occurred. Some transmission latency will occur, and three extra tests are required in the modified algorithm to check whether the algorithm is in its initial pass.

In summary, if the modified SPIHT is selected by the simple test, then the algorithm of [1, 3] for $p = 0$ becomes

1. **Initialization:** use T_0' as the threshold; otherwise outputs are as before.

2. **Sorting:**

   2.1 for each entry (i, j) in the list of insignificant pixels do:

   2.1.1 output $S_0(i, j)$ as before;

2.1.2 If $S_0(i,j) = 1$ as before; But also if $c_{i,j} \geq T_0$ then output 1, otherwise output 0; then move (i,j) to the list of significant pixels as before.

2.2 For each entry (i,j) in the list of insignificant sets do:

2.2.1 If the entry is the set of all descendants of (i,j), $D(\dot{})$, then output $S_0(D(i,j))$ as before; If $S_0(D(i,j)) = 1$, then

- Proceed as before by testing each coefficient of direct descendants indexed as (k,l). However, should $S_0(k,l) = 1$, then if $c_{k,l} \geq T_0$ then output 1, otherwise output 0; proceed as before.
- Otherwise proceed as before.

2.2.2 Remains unchanged.

3. **Refinement:** Remains unchanged.

4. **Quantization update** Remains unchanged.

3 Results and Conclusion

In Table 1, for those 512×512 standard grey-scale test images that satisfied the simple test, the SPIHT and modified SPIHT bits/pel (bpp) are compared. Clearly, the percentage improvement increases with reduction of PSNR. We also reduced the dynamic range of the pixels in each image by subtracting 128 from the each pixel, in the manner of the IW44 algorithm [5], to achieve a low-complexity approximation to zero meaning. The result was a reduction from eight to six of the images ('bridge', 'barbara', and 'man' are now unmodified, with 'crowd' becoming modified) that passed the initial simple test, but gains in bit-rate for the six that are similar to the results of Table 1.

This letter has recommended a simple modification to the SPIHT quantizer, which can be effected in hardware, and leads in many cases to an improved bit-rate, without any distortion penalty. The improvement is the result of removing one round of embedded bit-map transmission. It is thought that that the technique may be applicable to similar algorithms such as EBCOT [6].

References

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[5] L. Bottou, P. Haffner, P. G. Howard, P. Simard, B. Yoshua, and Y. LeCun. High quality document image compression with DjVu. *Journal of Electronic Imaging*, 7(3):410–425, 1998.

[6] D. Taubman. High performance scalable image compression with EBCOT. *IEEE Transactions on Image Processing*, 9(4):1158–1170, 2000.

List of Tables

Image	SPIHT (bpp)	Modified (bpp)	Improvement (%)	PSNR (dB)
Airfield	not modified			
Airplane	0.0345	0.0295	14.3	21.78
	0.1394	0.1337	4.2	27.71
	0.4974	0.4913	1.2	34.54
Barbara	0.0326	0.0273	16.2	20.62
	0.1892	0.1831	3.2	25.31
	0.7965	0.7904	0.7	32.76
Bridge	not modified			
Camera	not modified			
Couple	0.0323	0.0268	16.9	21.54
	0.1492	0.1428	4.3	26.17
	0.7050	0.6988	0.9	32.81
Crowd	not modified			
Harbour	0.0314	0.0257	18.3	20.88
	0.2069	0.2005	3.1	25.08
	1.004	0.9979	0.6	32.58
Lax	0.0323	0.0273	17.6	20.34
	0.2280	0.2222	2.5	23.97
	1.3337	1.3306	0.5	30.72
Lenna	0.0320	0.0266	19.1	23.04
	0.1044	0.0980	6.1	28.50
	0.3937	0.3876	1.6	34.62
Man	0.0317	0.0264	18.8	21.91
	0.1306	0.1245	4.7	26.52
	0.6744	0.6683	0.9	33.08
Baboon	0.0310	0.0245	21.1	19.15
	0.2740	0.2676	2.3	22.69
	1.5198	1.5137	0.4	30.33
Peppers	not modified			

Table 1: Comparative bit-rates for standard test images