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Semifluxons with a hump in a 0–π Josephson junction

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Abstract

We discuss the sine-Gordon equation describing the phase difference in a 0–π Josephson junction. Via phase plane analysis, it is shown that the time-independent equation can have semifluxons with a hump. A stability analysis to the semifluxons is performed. It is shown numerically that the presence of defects can stabilize the semifluxons.

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One of the consequences of non-s-wave superconductivity is the possibility of the spontaneous formation of a half-vortex which is attached to the point of discontinuity in a 0–π Josephson junction. This half-vortex has been studied theoretically by several authors [1–5]. The spontaneous fluxons are used to probe the symmetry of a superconducting gap. It has been conjectured that these half-vortices can also be utilized in superconducting memory and logic devices [3] and accurate measurements of the Josephson penetration length of a superconductor [4].

The electrodynamics of the long Josephson junction is described by the sine-Gordon equation [5]

$$\phi_{xx} - \phi_{tt} = \theta(x)\sin\phi - \gamma + \alpha\phi_t, \tag{1}$$

where $\phi(x,t)$ denotes the Josephson phase difference of the junction, $\alpha > 0$ is the damping coefficient due to current electron flow across the junction and $\gamma > 0$ is the applied bias current. The function θ takes the value ± 1 representing the alternating sign of the critical current. In the case of one discontinuity, $\theta = 1$ for $x < 0$ and -1 for $x > 0$.

Semifluxon solutions of (1) satisfy the condition $|\phi(\infty) - \phi(-\infty)| = \pi$. When $\gamma = 0$ there is a unique semifluxon solution (mod 2π) given by

$$\phi_0(x) = \begin{cases} 4\arctan\exp(x - x_0), & x < 0, \\ 4\arctan\exp(x + x_0) - \pi, & x > 0, \end{cases} \tag{2}$$

where $x_0 = \ln(\sqrt{2} + 1)$. By considering time independent solutions of (1), i.e. looking for solutions of the equation

$$\phi_{xx} = \theta(x)\sin\phi - \gamma, \tag{3}$$

it will be shown that for $\gamma \neq 0$, the semifluxon is not unique. There are semifluxons with a hump.

In [6], the semifluxons of Eq. (3) are constructed using phase portrait analysis. The phase portrait is constructed from two phase planes, namely the one with $\theta = 1$ and the other one with $\theta = -1$. Solutions of Eq. (3) are then suitable combinations of orbits of the two phase planes. The semifluxons with a hump can be constructed when $\gamma \neq 0$, because the heteroclinic connections of the sine-Gordon equation break into homoclinic connections. For a more detailed explanation we refer to [6].

When $\gamma \in (0, \gamma^* = 2/\sqrt{4 + \pi^2})$, there are three semifluxons that can be constructed. The plot of the semifluxons with a hump as functions of x is presented in Fig. 1. For simplicity, we denote semifluxon with the higher hump as $SF^{(2)}$ and the other as $SF^{(3)}$. The number of kinks decreases to two when $\gamma \in (\gamma^*, \gamma_c = 2/\pi)$. The semifluxon of type 2 disappears for $\gamma > \gamma^*$. For $\gamma > \gamma_c$, there is no semifluxon solution of Eq. (3) [6].

We are interested in the stability of $SF^{(2)}$ and $SF^{(3)}$. We determine numerically the largest eigenvalue of the

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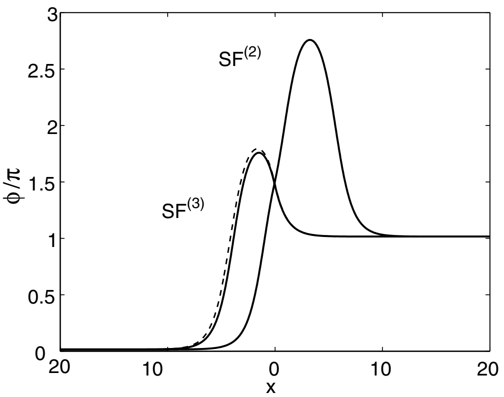

Fig. 1. A plot of the two semifluxons with a hump (solid-lines) for $\gamma = 0.05$. The dashed-line is the steady state of an evolution of the $SF^{(3)}$ in presence of a defect (see the text).

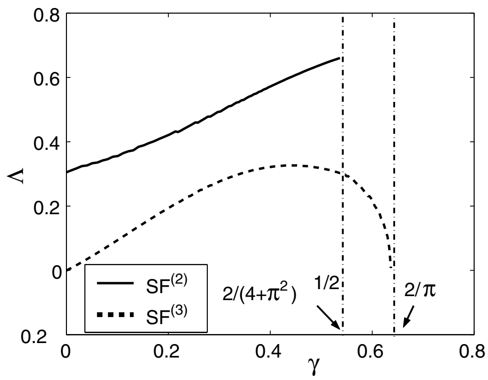

Fig. 2. The largest eigenvalues of the semifluxons are drawn as functions of γ.

semifluxons. By taking the spectral ansatz $\phi(x,t) = u(x)e^{\lambda t}$ and writing $\Lambda = \lambda^2 + \lambda\alpha$, we obtain from (1) the eigenvalue equation $u_{xx} - \theta(x)u\cos\psi_0 = \Lambda u$ where ψ_0 is the semifluxons of type 2 or 3. The spectral parameter Λ is presented in Fig. 2.

It is clear then that the solutions are not stable because there is at least a positive eigenvalue for all γ. Note that $SF^{(3)}$ tends to neutral stability in the limits $\gamma \to 0$ and $\gamma \to 2/\pi$. The neutral stability when $\gamma = 2/\pi$ is not surprising if one realizes that in the phase portrait, $SF^{(3)}$ coincides with the well-known semifluxon. According to Ref. [3], the largest eigenvalue of the semifluxons is 0 at that situation.

The evolution of the semifluxons is simulated by taking semifluxons with a hump obtained from (3) as initial solutions for (1). We impose $\phi_x(x = \pm L, t) = \phi_t(x, t = 0) = 0$ and take $L = 20$. In Fig. 3, we present the evolution of the $SF^{(3)}$ shown in Fig. 1.

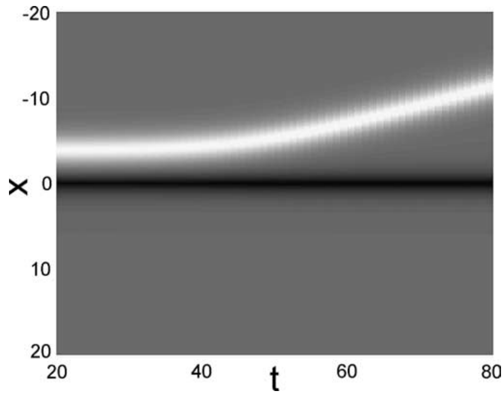

Fig. 3. The evolution of the $SF^{(3)}$ shown in Fig. 1 with $\alpha = 0.1$ in terms of ϕ_x. A 2π-fluxon (bright line) is driven by the applied bias current away from the semifluxon (dark line).

We observe numerically that a small and localized perturbation of the system can stabilize a semifluxon with a hump. The perturbation is of the form $\epsilon v \sin\phi$ that is added to the right hand side of (1). A localized defect in the junction can be represented by this perturbation with $|\epsilon| \ll 1$ representing the depth of the defect and $v = 1$ at a certain interval and 0 otherwise representing the localization of the defect.

We have redone the simulation presented in Fig. 3 in presence of a defect at interval (−5,−3) with the depth $\epsilon = -0.2$. The steady state (the dashed-line in Fig. 1) is slightly different from the initial one due to the perturbation term. Position, width and depth of the defect influence the final form of the semifluxon.

To conclude, we have discussed numerically the stability of semifluxons with a hump. Even though they are unstable, defects can stabilize the semifluxon of type 2. For future investigation, we want to prove analytically the stabilization of the semifluxons.

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