SEMIFLUXONS IN A LONG JOSEPHSON JUNCTION WITH A π-DISCONTINUITY POINT *

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We consider the sine-Gordon equation describing the phase difference of a Josephson junction with a π-discontinuity point. It is known that the time-independent equation can have non-monotone semifluxons. A stability analysis to the semifluxons is performed. It is shown numerically that the presence of defects can stabilize the semifluxons.

1. Introduction

Superconductors are characterized by the phase coherence of the cooper-pair condensate. Josephson (1962) first pointed out that it is possible for Cooper pairs to flow through a thin insulating barrier between two superconductors. The intrinsic anisotropy of unconventional superconductivity offers the possibility of phase biasing of such Josephson junctions. These junctions are characterized by an intrinsic phase-shift of π in the current-phase relation or, in other words, an effective negative critical current. A predominant $d_{x^2-y^2}$ pairing symmetry in high-T_c superconductors enables the possibility to bias parts of the circuit with a phase of π.[1] An example is a $YBa_2Cu_3O_7$−Au−Nb ramp-type corner junction.[2] A sketch of this corner junction is presented in Fig. 1. These structures, of which neighboring facets in a Josephson junction have opposite sign of the critical current, present intriguing phenomena such as the intrinsic frustration of the Josephson phase over the junction and the spontaneous generation of fractional magnetic flux near the corners. The presence of fractional flux, or semifluxons, has been considered before by several authors.[3,4,5,6]

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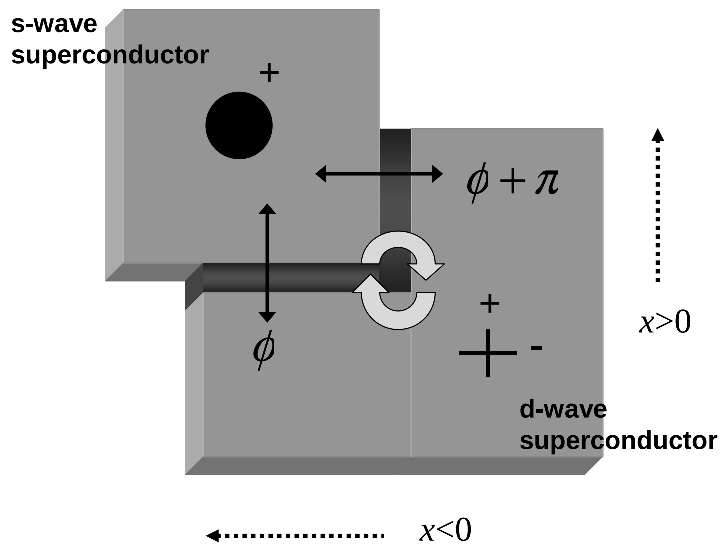

Figure 1. A sketch of a corner junction made by two superconductors, i.e., an s-wave superconductor (low-T_c superconductor) and a d-wave superconductor (high-T_c superconductor). A circulating current with a half-flux quantum, or semifluxon, is created and attached to the discontinuity point.

The spontaneous fluxons can be used to probe the symmetry of a superconducting gap. It has been conjectured that these half-flux quanta can also be utilized in superconducting memory and logic devices and accurate measurements of the Josephson penetration length of a junction.[4,5]

2. Mathematical model and numerical scheme

The electrodynamics of the long Josephson junction is described by the sine-Gordon equation[6]

$$\phi_{xx} - \phi_{tt} = \theta(x) \sin\phi - \gamma + \alpha\phi_t, \qquad (1)$$

where $\phi(x,t)$ denotes the Josephson phase difference of the junction, $\alpha > 0$ is the damping coefficient due to current electron flow across the junction and $\gamma > 0$ is the applied bias current. The function θ takes the value ± 1 representing the alternating sign of the critical current. In the case of one discontinuity, $\theta = 1$ for $x < 0$ and -1 for $x > 0$.

To perform simulations of Eq. (1), we use an implicit finite difference scheme where the differentiations are replaced by the differences such that at time $t = n\Delta t$ and $x = j\Delta x$

$$\begin{aligned}
&\phi_{tt} \approx [\phi_j^n - 2\phi_j^{(n-1)} + \phi_j^{(n-2)}]/\Delta t^2,\\
&\phi_{xx} \approx [\phi_{(j+1)}^n - 2\phi_j^n + \phi_{(j-1)}^n]/\Delta x^2,\\
&\phi_t \approx [\phi_j^n - \phi_j^{(n-1)}]/\Delta t,\\
&\sin\phi \approx \sin\phi_j^{(n-1)},
\end{aligned}$$

with the boundary conditions $\phi_x = 0|_{x=\pm L}$ and $\phi_t = 0|_{t=0}$. In the calculation, we take $\Delta t = \Delta x = 0.1$ and $L = 20$.

3. Semifluxons of a 0-π Josephson junction

Semifluxon solutions of Eq. (1) satisfy the condition $|\phi(\infty) - \phi(-\infty)| = \pi$. When $\gamma = 0$ there is a unique semifluxon solution (mod 2π) given by

$$\phi_0(x) = \begin{cases} 4\arctan\exp(x - x_0),\ x < 0, \\ 4\arctan\exp(x + x_0) - \pi,\ x > 0, \end{cases} \tag{2}$$

where $x_0 = \ln(\sqrt{2} + 1)$. This semifluxon is a combination of two 2π-fluxons of the sine-Gordon equation with $\theta \equiv 1$ and $\theta \equiv -1$.

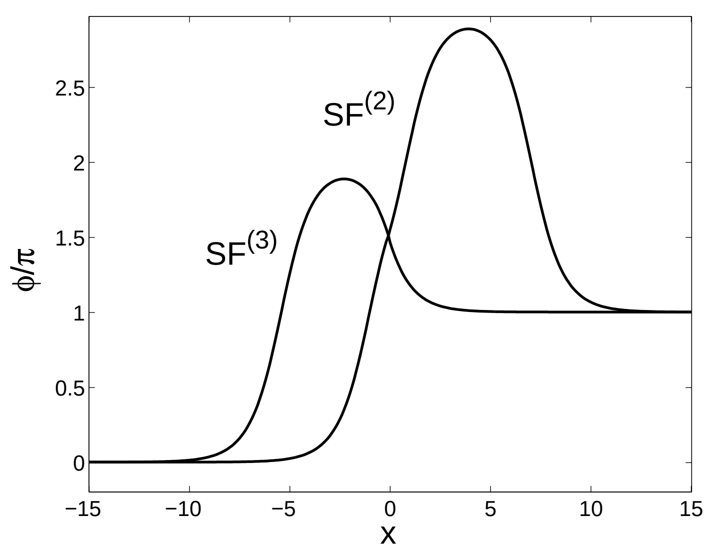

Figure 2. The plot of the two non-monotone semifluxons for $\gamma = 0.01$. The solution with the higher and lower hump is denoted by $SF^{(2)}$ and $SF^{(3)}$, respectively.

By considering time independent solutions of Eq. (1), i.e. looking for solutions of the equation

$$\phi_{xx} = \theta(x) \sin \phi - \gamma, \tag{3}$$

it has been shown previously, see [7], that for $\gamma \neq 0$, the semifluxon is not unique. There are two non-monotone semifluxons.

These semifluxons can be seen to exist using phase portrait analysis.[7] The phase portrait is obtained from two phase planes, namely the one with $\theta \equiv 1$ and the other one with $\theta \equiv -1$. Solutions of Eq. (3) are then suitable combinations of orbits of the two phase planes. The non-monotone semifluxons can be constructed for $\gamma \neq 0$, because the heteroclinic connections of the sine-Gordon equation break into homoclinic connections. For a more detailed explanation we refer to [7].

When $\gamma \in (0, \gamma^* = 2/\sqrt{4+\pi^2})$, there are three semifluxons. The plot of the non-monotone semifluxons as functions of x is presented in Fig. 2. The number of kinks decreases to two when $\gamma \in (\gamma^*, \gamma_c = 2/\pi)$. The semifluxon $\mathrm{SF}^{(2)}$ does not exist for $\gamma > \gamma^*$. For $\gamma > \gamma_c$, Eq. (3) does not admit a semifluxon solution.[7]

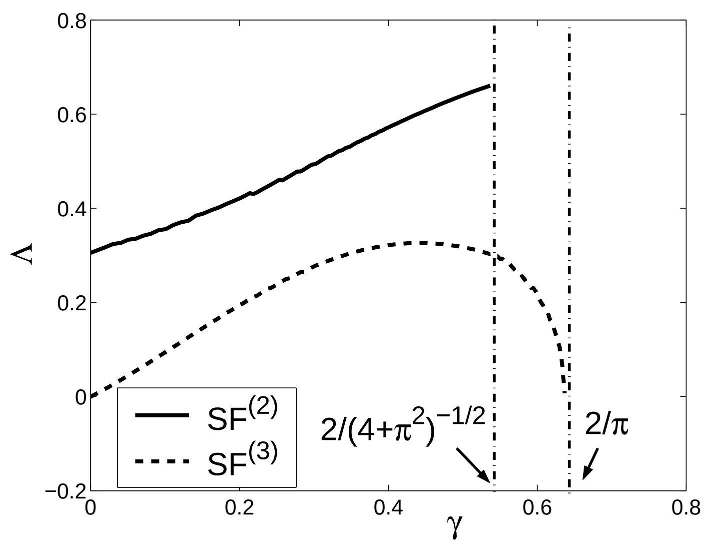

Figure 3. The largest eigenvalues of the semifluxons are drawn as a function of γ.

4. Stability and stabilization of the semifluxons

The monotone semifluxon Eq. (2) is stable.[8] This solution seems to be a global attractor. We are interested in the stability of $SF^{(2)}$ and $SF^{(3)}$. We determine numerically the largest eigenvalue of the semifluxons. By taking the spectral ansatz $\phi(x,t) = u(x)e^{\lambda t}$ and writing $\Lambda = \lambda^2 + \lambda\alpha$, we obtain from Eq. (1) the eigenvalue equation

$$u_{xx} - \theta(x)u\cos\psi_0 = \Lambda u$$

where ψ_0 is the semifluxons of type 2 or 3. The largest eigenvalue is presented in Fig. 3. It is clear then that the solutions are not stable because there is at least a positive eigenvalue for all γ.

In the limit $\gamma \to 0$, the largest eigenvalue of $SF^{(2)}$ tends to

$$\Lambda = (\sqrt{5} - 1)/4 \approx 0.31.$$

This can be computed explicitly.[8] At $\gamma = 0$, the $SF^{(2)}$ solution is a 3π-kink (mod 2π) given by

$$\phi_0(x) = \begin{cases} 4\arctan\exp\left(x + x_0\right),\ x < 0, \\ 4\arctan\exp\left(x - x_0\right) + \pi,\ x > 0. \end{cases} \tag{4}$$

The soliton $SF^{(3)}$ tends to neutral stability in the limits $\gamma \to 0$ and $\gamma \to 2/\pi$. The neutral stability at $\gamma = 2/\pi$ is not surprising if one realizes that in the phase portrait, $SF^{(3)}$ coincides at this point with the monotone semifluxon which has 0 as its largest eigenvalue.[4] A time evolution of $SF^{(3)}$ shown in Fig. 2 is presented in Fig. 4(a).

We observe numerically that a small and localized perturbation of the system can stabilize a non-monotone semifluxon. The perturbation we consider is of the form $\delta(a)\sin\phi$ that is added to the right hand side of Eq. (1) where $\delta(a)$ is an approximation of the δ-distribution localized at $x = a$. Experimentally, a defect can be made by a localized-heating using an ion-beam.[9]

We have repeated the above simulation shown in Fig. 4(a) in presence of a defect. The defect is placed such that $\delta = -2$ at the interval $[-5.10, -5.05]$. The stabilization of the solution by the defect is presented in Fig. 4(b). The steady state is slightly different from the initial solution. Position and depth of the defect influence the final form.

5. Conclusions

To conclude, we have shown numerically the instability of non-monotone semifluxons in the time-independent equation. Even though they are un-

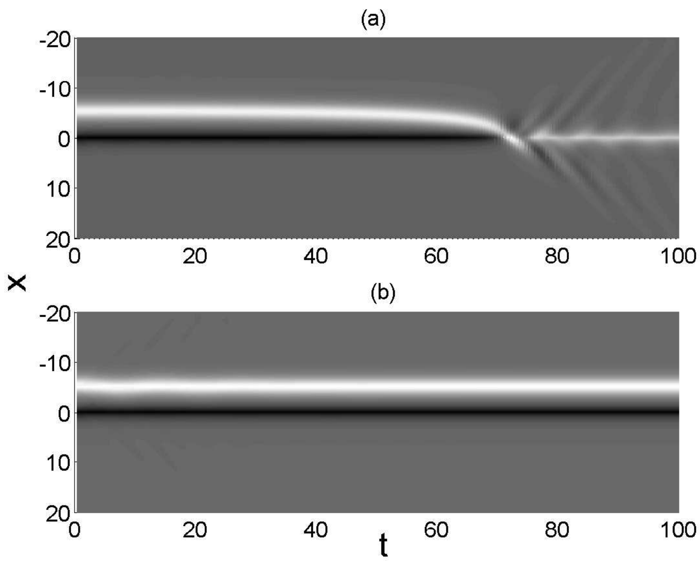

Figure 4. Evolution of the solution $SF^{(3)}$ shown in Fig. 2 with $\alpha = 0.01$ is presented in terms of the magnetic field ϕ_x. (a) The solution is unstable. We see the collision of a 2π-fluxon (bright line before $t \approx 75$) and an anti-semifluxon (dark line) producing a semifluxon (bright line after $t \approx 80$). (b) The stabilization of the solution by a defect.

stable, defects can stabilize them. To prove analytically the stabilization of the semifluxons by the defect and more specifically to characterize the properties of the defect in order to do so, is work in progress.

References

1. C. C. Tsuei and J. R. Kirtley, *Rev. Mod. Phys.* **72**, 969 (2000).
2. H. Hilgenkamp, Ariando, H. J. H. Smilde, *et al.*, *Nature* **422**, 50 (2003).
3. J. H. Xu, J. H. Miller, Jr. and C. S. Ting, *Phys. Rev.* **B51**, 11958 (1995).
4. A. B. Kuklov, V. S. Boyko and J. Malinsky, *Phys. Rev.* **B51**, 11965 (1995); **B55**, 11878 (1997).
5. T. Kato and M. Imada, *J. Phys. Soc. Jpn.* **66**, 1445 (1997).
6. E. Goldobin, D. Koelle and R. Kleiner, *Phys. Rev.* **B66**, 100508 (2002).
7. H. Susanto, S. A. van Gils, T. P. P. Visser, *et al.*, *Phys. Rev.* **B68**, 104501 (2003).
8. G. Derks, A. Doelman, S. A. van Gils and H. Susanto, in preparation.
9. D. Koelle, private communications.