This is a simple test worksheet for the QuaternionAlgebra package.

Steve Sangwine
February 2005

```
> restart;
> with(QuaternionAlgebra);
```

[*Argument*, *Axis*, *Conjugate*, *Exponential*, *Inverse*, *Involute*, *MakePureQuaternion*, *MakeQuaternion*, *Modulus*, *Par*, *Perp*, *QNorm*, *S*, *Scalar*, *ScalarProduct*, *Unit*, *V*, *VectorProduct*, *X*, *Y*, *Z*, *Zero*, *e0*, *e1*, *e2*, *e3*]

```
> with(LinearAlgebra):
> P:=MakeQuaternion(p);
```

$$P := \begin{bmatrix} p_w & p_x & p_y & p_z \\ -p_x & p_w & -p_z & p_y \\ -p_y & p_z & p_w & -p_x \\ -p_z & -p_y & p_x & p_w \end{bmatrix}$$

```
> Q:=MakeQuaternion(q);
```

$$Q := \begin{bmatrix} q_w & q_x & q_y & q_z \\ -q_x & q_w & -q_z & q_y \\ -q_y & q_z & q_w & -q_x \\ -q_z & -q_y & q_x & q_w \end{bmatrix}$$

```
> P.Q;
```

$[p_w q_w - p_x q_x - p_y q_y - p_z q_z, p_w q_x + p_x q_w + p_y q_z - p_z q_y, p_w q_y - p_x q_z + p_y q_w + p_z q_x, p_w q_z + p_x q_y - p_y q_x + p_z q_w]$

$[-p_x q_w - p_w q_x + p_z q_y - p_y q_z, p_w q_w - p_x q_x - p_y q_y - p_z q_z, -p_x q_y - p_w q_z - p_z q_w + p_y q_x, p_w q_y - p_x q_z + p_y q_w + p_z q_x]$

$[-p_y q_w - p_z q_x - p_w q_y + p_x q_z, p_w q_z + p_x q_y - p_y q_x + p_z q_w, p_w q_w - p_x q_x - p_y q_y - p_z q_z, -p_x q_w - p_w q_x + p_z q_y - p_y q_z]$

$[-p_x q_y - p_w q_z - p_z q_w + p_y q_x, -p_y q_w - p_z q_x - p_w q_y + p_x q_z, p_w q_x + p_x q_w + p_y q_z - p_z q_y, p_w q_w - p_x q_x - p_y q_y - p_z q_z]$

```
> Scalar(P),X(P),Y(P),Z(P);
```

$$p_w, p_x, p_y, p_z$$

```
> QNorm(P);
```

$$p_w^2 + p_x^2 + p_y^2 + p_z^2$$

```
> Modulus(P);
```

$$\sqrt{p_w^2 + p_x^2 + p_y^2 + p_z^2}$$

```
> Conjugate(P);
```

$$\begin{bmatrix} p_w & -p_x & -p_y & -p_z \\ p_x & p_w & p_z & -p_y \\ p_y & -p_z & p_w & p_x \\ p_z & p_y & -p_x & p_w \end{bmatrix}$$

```
> simplify(Unit(P));
```

$$\left[\frac{p_w}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_x}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_y}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_z}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}\right]$$

$$\left[-\frac{p_x}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_w}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, -\frac{p_z}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_y}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}\right]$$

$$\left[-\frac{p_y}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_z}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_w}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, -\frac{p_x}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}\right]$$

$$\left[-\frac{p_z}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, -\frac{p_y}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_x}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}, \frac{p_w}{\sqrt{p_w^2+p_x^2+p_y^2+p_z^2}}\right]$$

```
> Axis(P);
```

$$\frac{\begin{bmatrix} 0 & p_x & p_y & p_z \\ -p_x & 0 & -p_z & p_y \\ -p_y & p_z & 0 & -p_x \\ -p_z & -p_y & p_x & 0 \end{bmatrix}}{\sqrt{p_x^2+p_y^2+p_z^2}}$$

```
> Argument(P);
```

$$\arctan(\sqrt{p_x^2+p_y^2+p_z^2}, p_w)$$

```
> Inverse(P);
```

$$\frac{\begin{bmatrix} p_w & -p_x & -p_y & -p_z \\ p_x & p_w & p_z & -p_y \\ p_y & -p_z & p_w & p_x \\ p_z & p_y & -p_x & p_w \end{bmatrix}}{p_w^2 + p_x^2 + p_y^2 + p_z^2}$$

```
> S(P),V(P);
```

$$\begin{bmatrix} p_w & 0 & 0 & 0 \\ 0 & p_w & 0 & 0 \\ 0 & 0 & p_w & 0 \\ 0 & 0 & 0 & p_w \end{bmatrix}, \begin{bmatrix} 0 & p_x & p_y & p_z \\ -p_x & 0 & -p_z & p_y \\ -p_y & p_z & 0 & -p_x \\ -p_z & -p_y & p_x & 0 \end{bmatrix}$$

```
> ScalarProduct(P,Q),VectorProduct(V(P),V(Q));
```

$$p_w q_w + p_x q_x + p_y q_y + p_z q_z, \begin{bmatrix} 0 & p_y q_z - p_z q_y & p_z q_x - p_x q_z & p_x q_y - p_y q_x \\ -p_y q_z + p_z q_y & 0 & -p_x q_y + p_y q_x & p_z q_x - p_x q_z \\ -p_z q_x + p_x q_z & p_x q_y - p_y q_x & 0 & -p_y q_z + p_z q_y \\ -p_x q_y + p_y q_x & -p_z q_x + p_x q_z & p_y q_z - p_z q_y & 0 \end{bmatrix}$$

```
> U:=simplify(Unit(MakePureQuaternion(u)));
```

$$U := \begin{bmatrix} 0 & \frac{u_x}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & \frac{u_y}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & \frac{u_z}{\sqrt{u_x^2 + u_y^2 + u_z^2}} \\ -\frac{u_x}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & 0 & -\frac{u_z}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & \frac{u_y}{\sqrt{u_x^2 + u_y^2 + u_z^2}} \\ -\frac{u_y}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & \frac{u_z}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & 0 & -\frac{u_x}{\sqrt{u_x^2 + u_y^2 + u_z^2}} \\ -\frac{u_z}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & -\frac{u_y}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & \frac{u_x}{\sqrt{u_x^2 + u_y^2 + u_z^2}} & 0 \end{bmatrix}$$

```
> P_par:=Par(P,U):
> P_perp:=Perp(P,U):
> simplify(ScalarProduct(P_par,P_perp));
```

$$0$$

```
> P_par + P_perp;
```

$$\begin{bmatrix} p_w & p_x & p_y & p_z \\ -p_x & p_w & -p_z & p_y \\ -p_y & p_z & p_w & -p_x \\ -p_z & -p_y & p_x & p_w \end{bmatrix}$$

```
> ScalarProduct(V(P),VectorProduct(V(P),V(Q)));
```

$$p_x (p_y q_z - p_z q_y) + p_y (p_z q_x - p_x q_z) + p_z (p_x q_y - p_y q_x)$$

```
> simplify(%);
```

$$0$$