PROCESSING OF COLOUR IMAGES USING HYPERCOMPLEX NUMBERS

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Abstract

We present in this poster overviews of some recent and ongoing work on processing of colour images using hypercomplex numbers (or quaternions).
Since 1996, Fourier transforms and basic linear image processing operations such as correlation have been a focus for applying hypercomplex numbers to colour images. More recently, work has also included development of new image filters based on convolution with hypercomplex masks.
We present here a broad overview of our work.

Quaternions

- General Form $Q = a + ib + jc + kd$
- Scalar part $S[Q] = a$, Vector part $V[Q] = ib+jc+kd$
- If $S[Q] = 0$, then Q is a *pure* quaternion.
- Multiplication of pure quaternions,

$$S[PQ] = -P.Q,\ V[PQ] = P \times Q,\ PQ \neq QP.$$

- Pure quaternions can be used to represent RGB colour space with the origin shifted to mid-grey, ie.

$$Q = i(R - O) + j(G - O) + k(B - O).$$

- Polar Form $Q = |q| \exp \boldsymbol{\mu}\phi$ (cf. complex numbers)
- Rotating by ϕ about the axis μ,

$$Q' = \exp(\boldsymbol{\mu}\phi/2) Q \exp(-\boldsymbol{\mu}\phi/2).$$

- If $\boldsymbol{\mu}$ is the colour intensity axis, $\frac{1}{\sqrt{3}}(i + j + k)$, then the hue of the colours represented by Q and Q' differ by ϕ.

Hypercomplex numbers

Comparison between scalar numbers, complex numbers and quaternions:

Dimensions	Number	General Form	Additional Details
1	Scalar	a	–
2	Complex	$a + ib$	$i^2 = -1$
4	Quaternion	$a + ib + jc + kd$	$i^2 = j^2 = k^2 = -1$ $ij = k, jk = i, ki = j$ $ji = -k, kj = -i, ik = -j$

The complex numbers cannot be ordered. The quaternion numbers cannot be ordered, *and* multiplication is not commutative.

Hypercomplex chromatic edge detector[1]

- Chromatic edge: sharp transition in hue or saturation. Achromatic edge: sharp intensity transition alone.
- Chromatic edge enhancement by hypercomplex convolution $L * Q_{Im} * R$.
- $L = \frac{1}{6}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline A & A & A \\ \hline \end{array}$ $R = \frac{1}{6}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline \bar{A} & \bar{A} & \bar{A} \\ \hline \end{array}$ where $\boldsymbol{\mu} = \frac{(i+j+k)}{\sqrt{3}}$ and $A = \exp \boldsymbol{\mu}(\pi/2)$.
- Achromatic edges and homogenous coloured regions in the original image appear grey in the filtered image, but chromatic edges have a defined hue.

Lena image (left); Results of chromatic edge detection (right). (The result image is a colour image with grey pixels everywhere except at colour edges where colour can be seen.)

Colour Sensitive Edge Detection

Hypercomplex image filters have also been designed for the purpose of colour-sensitive edge detection. For example,

- detecting $C_1 \longrightarrow C_2$-coloured edges, where C_1 and C_2 are user-specified.

Left to right: original image; edge detected image; binary edge image.

- detecting all edges of a C_1-coloured object, where C_1 is user-defined.

Left to right: original image; edge detected image; binary edge image.

Preliminary work in these areas has been submitted to EUSIPCO 2000 and ICIP 2000 but is yet to be published.

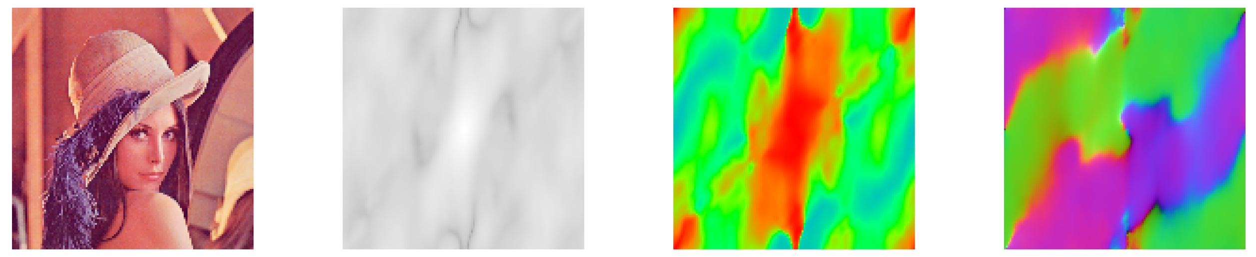
Hypercomplex Correlation[3]

Correlation is another basic operation in signal and image processing and it is possible to generalize the complex formula to hypercomplex signals and images. We use the standard definition of cross-correlation of two images (see for example: Gonzalez and Woods), but both images and the result are hypercomplex, and the conjugate is hypercomplex:

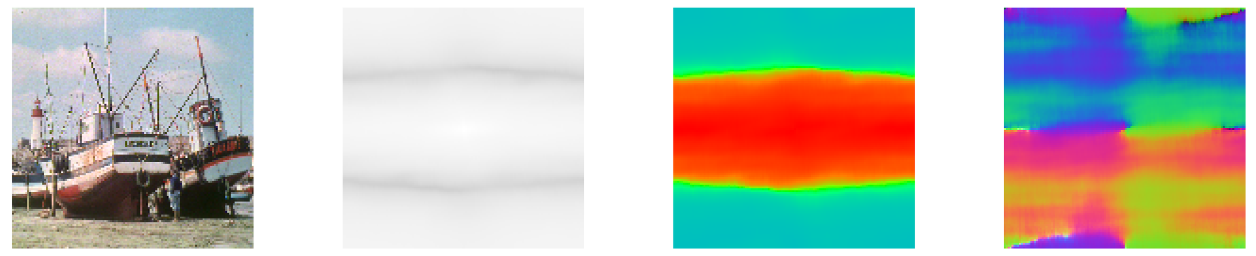
$$r(n, m) = \sum_{p=0}^{N-1} \sum_{q=0}^{M-1} f(p, q)\overline{g(p - n, q - m)}$$

Visualization utilizes polar form of quaternion result.

Results – autocorrelation of natural images



Autocorrelation of the 'Lena' image (128 × 128 pixels). Left to right: original image, modulus, phase, eigenaxis.



Autocorrelation of the 'Boat' image (128 × 128 pixels). Left to right: original image, modulus, phase, eigenaxis. (Original images from the USC-SIPI image database.)

Hypercomplex Fourier transform[2]

$$F(u, v) = S \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{-\boldsymbol{\mu}2\pi\left(\frac{mu}{M} + \frac{nv}{N}\right)} f(m, n)$$

$$f(m, n) = S \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} e^{\boldsymbol{\mu}2\pi\left(\frac{mu}{M} + \frac{nv}{N}\right)} F(u, v)$$

where, $S = 1/\sqrt{MN}$, and $\boldsymbol{\mu}$ is *any* unit pure quaternion. (We can also define a right handed version of this transform, which is very closely related.)

References

[1] S. J. Sangwine. Colour image edge detector based on quaternion convolution. *Electronics Letters*, 34(10):969–971, 14 May 1998.

[2] S. J. Sangwine and T. A. Ell. The discrete Fourier transform of a colour image. In J. M. Blackledge, editor, *Second IMA Conference on Image Processing: Mathematical Methods, Algorithms and Applications, De Montfort University, Leicester, UK 22-25 September 1998*, pages –, Chichester, 1999. Horwood Publishing for Institute of Mathematics and its Applications. In Press.

[3] S. J. Sangwine and T. A. Ell. Hypercomplex auto- and cross-correlation of color images. In *IEEE International Conference on Image Processing (ICIP99)*, pages 319–322 (missing one page), Kobe, Japan, October 24-28 1999. Full 5pp on CD-ROM proceedings, ISBN 0-7803-5470-2.

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