The Royal Academy of Engineering

Global Research Awards

Quaternion signal & image processing

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Seconded to: Laboratoire des Images et des Signaux, Institut National Polytechnique de Grenoble, France

Outline: I spent 7 months seconded to a laboratory in Grenoble from January to July 2005 working with a collaborator that I had known for almost 4 years previously. Our mutual interest is in signal and image processing using quaternion algebra. Quaternions are 4-dimensional hypercomplex numbers, usually represented in hypercomplex form as: $q = w + \boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z$. We use them to represent the samples of vector signals (example: colour images with RGB components to each pixel). The algebra of the quaternions neatly handles geometrical operations in 3-space such as rotations, dilations, affine transformations etc.

Outcomes from the secondment:

- A simple algorithm for computing the singular value decomposition of a quaternion matrix [1].
- A complexified quaternion Fourier transform [2].
- Determination of the square roots of -1 in complexified quaternions [3] (utilised in [2]).
- A very comprehensive Matlab toolbox for handling quaternion matrices [4]. (an unplanned result of the secondment, but probably the most significant.)
- Quaternion PCA (as yet unpublished).

Plus: a continuing collaboration with a prominent signal processing laboratory.

Language skills: Before going to France I improved my French from GCE 'O'-level standard by taking a 30-credit language module at Essex. On return I have taken another 30-credit module to second year Honours level. Although I was able to work with my collaborator in English, I gave seminars in French during my stay and attended weekly seminars presented in French. I needed to use French every day socially and outside the laboratory.

References

1. Sangwine, S. J. and Le Bihan, N., 'Quaternion singular value decomposition based on bidiagonalization to a real or complex matrix using quaternion Householder transformations', *Applied Mathematics and Computation*, 12pp., DOI :10.1016/j.amc.2006.04.032, in press, available online June 2006.
2. Said, S., Le Bihan, N. and Sangwine, S. J., 'Fast complexified quaternion Fourier transform', e-print *arXiv:math.NA/0603578*, 24 March 2006, available at http://www.arxiv.org/abs/math.NA/0603578.
3. Sangwine, S. J., 'Biquaternion (Complexified Quaternion) Roots of -1', *Advances in Applied Clifford Algebras*, 16, (1), February 2006, 63–68. DOI:10.1007/s00006-006-0005-8.
4. Sangwine, S. J. and Le Bihan, N., 'Quaternion Toolbox for Matlab', 2005. Available: http://qtfm.sourceforge.net/

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