

# Risk Sharing, (Over)Leverage, and the Dimensions of Competition

Tianxi Wang\*

## Abstract

The paper examines the leverage of financial intermediaries in a general equilibrium framework. The paper's approach to leverage is driven by risk sharing and captures two features: Debt serves to boost the return of equity and equity to "safe net" debt. The paper finds that if entrepreneurs compete only in the dimension of price, the equilibrium leverage rate is above the social best one; and if they compete in both price and investment scale, the two rates coincide, even under the market segmentation which drives financial intermediation. In the latter case, the credit market is cleared not by price, but by contract which specifies both price and scale. The paper argues that overleverage is more likely to occur where firms are small and dispersed while banks are large and concentrated, and shows that it can be rectified by proper capital adequacy regulation.

Key words: Risk Sharing    Leverage    Financial Intermediaries    Overleverage  
Contract-Taking Equilibrium    Capital Adequacy Regulation

---

\*University of Essex. Correspondence: Department of Economics, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ, Essex, UK. Email: wangt@essex.ac.uk. I am indebted to John Moore and Motty Perry for the long discussions with them. I also thank Sanjay Banerji, Sudipto Dasgupta, Leonardo Felli, Xuewen Liu, Wolfgang Pesendorfer, Francesco Squintani, and Asher Wolinsky, and the seminar audience of Univ. of Essex, Imperial College, ES 2010 World Congress, Hong Kong Univ. of Sci. and Tech., and Chinese Univ. of Hong Kong for the helpful comments.

# 1 Introduction

Leverage is an important ingredient of financial alchemy. For example, consider a project that requires investment of \$100 and returns \$101. If a private equity fund outlays the entire \$100 out of its own pocket, it earns a poor return rate of 1%; if it invests \$1 of its own fund and borrows \$99, then after paying back \$99 it earns \$2, with a shining return rate of 100%. Indeed, A study by British Private Equity & Venture Capital Association and Ernest & Young finds that half of the average return of the 14 biggest private equity deals realized in 2005-7 "came from the use of extra debt...."<sup>1</sup> Moreover, the recent crisis is closely tied up with banks being overleveraged.<sup>2</sup> How does a financial intermediary (**FI**) decides its leverage rate? Can the "invisible hand" take care of the decision? If it cannot, what market friction is to blame?

Literature used to approach leverage, implicitly or explicitly, through borrowing friction.<sup>3</sup> The paper presents a new approach which dispenses with borrowing friction and relies on risk sharing. Illustrate it with an example. There are one deep pocket and a lot of households. The deep pocket is risk neutral and has \$4. Each household has \$1 and is extremely risk averse, that is, when facing risk, he cares only for the worst outcome. Fund is either stored or invested in a risky project with gross return rate 0.8 or 1.4, each with half chance, so the expected rate is 1.1. Households, deterred by risk, choose to store their dollars. The deep pocket, as being risk neutral, puts all his capital in the risky project, and earns in expectation  $\$4 \times 1.1 = \$4.4$ , if he does not contract with households. However, he earns even more by levering with households' funds. Suppose he takes in one household's dollar and invests the entire \$5, the extra \$1 plus his own \$4, in the risky project. The investment returns  $\$5 \times 0.8 = \$4$  in the bad state and  $\$5 \times 1.4 = \$7$  in the good state. The household is satisfied with getting \$1 back in both states.

---

<sup>1</sup>See "Buy-out Profits Tied to Debt", *Financial Times*, January 15, 2009.

<sup>2</sup>See Brunnermeier(2009) and IMF *Global Financial Stability Report*, April, 2009.

<sup>3</sup>see, e.g., Geanakoplos and Polemarchakis (1986), Bernanke, Gertler and Gilchrist (1996), Geanakoplos (1997, 2003), Kiyotaki and Moore (1997), Adrian and Shin (2007, 2008), Brunnermeier and Pedersen (2008), Fostel and Geanakoplos (2008), Liu and Mello (2008), and Cao (2009). Note that in this literature, and in the present paper also, leverage means borrowing to expand asset scale. This meaning is different from what it means in the literature on capital structure which takes the asset side as given and focuses on debt-equity swap; for surveys of this literature see Harris and Raviv (1991) and Myers (2001).

The deep pocket thus earns, with this minor leverage,  $0.5 \times \$(4 - 1) + 0.5 \times \$(7 - 1) = \$4.5$ , \$0.1 more than he earns without any leverage. This extra \$0.1 is simply the difference between \$1.1, earned by investing the household's dollar in the project, and \$1, repaid to the household, which satisfies him since it is risk free. Therefore, the deep pocket can draw households' funds into the project and thereby earn profit margin \$0.1, by providing them with a risk free instrument. To provide it, the deep pocket gives seniority to the households' claims in the bad state; that is, the contract to them is debt, and his own fund forms the equity and acts as the cushion absorbing the loss to the debt holders in the bad state. The optimal amount of borrowing is \$16. At this level, the bad state earning,  $\$(4 + 16) \times 0.8 = \$16$ , exactly suffices to service the risk free debt; under, further borrowing still earns the profit margin of \$0.1; over, the debt is risky and scares away households given their extreme aversion to risk.

The approach is applied to examine the leverage of FIs in an economy where entrepreneurs have risky projects to be financed with the funds of households and deep pockets. Only deep pockets can directly invest in entrepreneurs and hence become FIs that bridge entrepreneurs and households; indeed, as illustrated above, they issue debt to households and put their own funds as equity, hence the leverage rate of FIs equal to the ratio of debt over equity. Entrepreneurs issue debt to FIs too. The price of debt, namely the promised rate of return, is actually paid only in the good state. In the bad state, entrepreneurs default and the rate of return decreases with the investment scale. The price, that is, the good state rate, must compensate the bad state rate, and is thus positively related to the scale: The bigger the scale, the lower the bad state rate; hence the higher the good state rate. Whether this positive relationship is taken into account is determined by the way in which entrepreneurs compete in the credit market and determines the efficiency of competitive equilibrium.

If entrepreneurs compete only in the dimension of price, the credit market is cleared by price in a usual general equilibrium way. In this *Price-Taking Equilibrium* (PTE), the relationship of price to scale is not taken into account: Each entrepreneur takes the price as given and ignores the effect that the expansion of his investment scale lowers the bad state rate, and thus pushes up the good state rate (namely the credit price), which harms all entrepreneurs. This externality induces entrepreneurs to over-demand credit, which in turn induces FIs levered above the social

best rate. The social best leverage is restored by the capital adequacy regulation that disallows FIs from being levered above the social best rate. Note that to enforce this regulation the government needs to know only as much as households, namely, the leverage rate of FIs, but nothing of entrepreneurs.

If entrepreneurs compete both in price and in investment scale, they take into account the positive relationship of price to scale, that is, an entrepreneur can ask for a lower price by committing to a smaller scale. The credit market is cleared not by price, but by contract which specifies a combination of price and scale. Given the equilibrium contract (namely the contract offered by all the other entrepreneurs), the representative entrepreneur offers the contract optimal to him, which will coincide with the equilibrium contract in equilibrium; and only if this contract prevails, the credit market clears. In this *Contract-Taking Equilibrium* (CTE), entrepreneurs internalize the adverse effect of investment scale on credit price. As a result, FIs are levered at the social best rate, *even though the economy is segmented*.

Therefore, *FIs are overleveraged, not because of market segmentation, but because of the friction that prevents entrepreneurs from competing in the dimension of investment scale*. The first instance of such friction is, obviously, that the investment scale is too costly to be observed or verified, that is, it is uncontractible. This is likely to occur where entrepreneurs' projects are too small to deserve being audited by a specialist. Second, where there are many small dispersed entrepreneurs but few large FIs, FIs could be so bureaucratic that the rate policies are decided in the upper and cannot be changed by the front line staff based on the investment scale. Third, the paper finds that deep pockets (namely FIs) earn more in the PTE than in the CTE. Therefore, they might collectively insist demanding a higher rate and decline to consider entrepreneurs' commitment. This is likely to occur, as before, where entrepreneurs are dispersed while FIs are concentrated. To sum up, the paper suggests that FIs are more likely to be overleveraged where firms are small and dispersed while banks are large and concentrated.

The paper offers support for EU proposed restrictions on the leverage of alternative investments funds (private equities and hedge funds etc.);<sup>4</sup> not only because alternative investment

---

<sup>4</sup>For the supportive side of the proposal, see, e.g, "Lessons from the Collapse of Bear Stearns" in *Financial Times* (14, 03, 2010) by John Cassidy; for the objecting side, see, e.g., "Regulate Providers of Debt Capital and

funds could be overleveraged, but also for a consideration of general equilibrium. The paper shows that leverage boosts the performance of equity. If the leverage of alternative investments is not restricted, then their equity is advantaged over the equity of commercial banks, since the latter, under capital adequacy regulation, cannot be as fully benefited from leverage as the former. To be fair, commercial banks are advantaged by state provided insurance, which makes the credit to them cheaper than that to alternative investments. However, this advantage of banks may not suffice to offset their disadvantage of restricted leverage. In this case, then, the management of banks are under the pressure of equity holders to catch up the better performance of alternative investments' equity, by e.g. setting up special investment vehicles (SIV) to be *actually* leveraged over the regulatory limit. In fact, SIV flourished in the US exactly during the period preceding the recent crisis when cheap credit was flooding thanks to the loose monetary policies and the vast inflow of foreign capital. In a word, if the leverage of alternative investments is not restricted, it may turn out difficult to regulate the leverage of commercial banks.

### ***Contributions and Relation with the Literature.***

The paper makes three main contributions.

First, it offers a new approach to endogenize leverage rate, an approach independent of borrowing friction. In the literature, leverage rate is either exogenously given (e.g. Kiyotaki and Moore (1997), Brunnermeier and Pedersen (2008), Liu and Mello (2008)) or driven by borrowing friction due to information asymmetry or limited commitment (e.g. Geanakoplos and Polemarchakis (1986), Bernanke, Gertler and Gilchrist (1996), Geanakoplos (1997, 2003), Adrian and Shin (2008), Fostel and Geanakoplos (2008), Cao (2009)). Furthermore, this new approach, compared to the existing ones, captures two features, that equity serves to "safe net" debt and that debt to boost the return of equity.<sup>5</sup> Especially, the former, though widely believed among

---

Get to the Problem's Root" in *Financial Times* (17, 05, 2010) by Ulf Axelson et. al.

<sup>5</sup>The idea that equity provides debt with safe net is also explored, e.g., by Gorton and Pinnacchi (1990) and Boot and Thakor (1993), who are concerned not with risk sharing but with informationally insensitive (or sensitive) securities.

practitioners, was not well captured in the academic circle, as Morrison and White (2005) noted.

Second, the paper uncovers a case of overborrowing that is of new nature, and based on it, offers a new justification for capital adequacy regulation. Literature has found various cases of overborrowing (or underborrowing); see, e.g., Gromb and Vayanos (2002), Fostel and Geanakoplos (2008), Lorenzoni (2008), Hombert (2009), and Korinek (2010), and see Wagner (2009) for a survey. In this literature, the externalities are driven by the feeding of market price (or the return rate) onto borrowing constraint or incentive compatibility constraint (in the manner of Anott, Greenwald and Stiglitz (1992)), an effect not internalized by decentralized agents.<sup>6</sup> In the present paper, since the borrowing friction of entrepreneurs towards households (i.e. the market segmentation) is not affected by anything, no such feeding presents itself, and the overborrowing is of new nature.

In the paper, the overborrowing problem is rectified by proper capital adequacy regulation. Although such regulation is prevalent and important, only little has been tried to rationalize it from primitives.<sup>7</sup> The most received reasoning for the regulation is based on proneness of banks to exploit demand deposit insurances, the provision of which is justified by Bryant (1980), Diamond and Dybvig (1983), and Gorton and Pinnachhi (1990). And Morrison and White (2005) examine capital requirements in relation to the reputation of the regulator. Complementing to the approaches above, the paper addresses the regulation through a market failure, with the capital markets on both the asset side and the liability side of FIs being endogenized.

Finally, the paper analyzes the concept of CTE to a financial market. Usually, when general equilibrium is concerned with, it means PTE. However, the paper suggests that PTE implicitly assumes the existence of friction that prevents economic agents from competing in other dimensions than price, and that where these dimensions exist, CTE is more relevant and appropriate

---

<sup>6</sup>In most cases, the higher the price (or the lower the return rate), the slacker the constraints. However, the opposite could happen, e.g. in Hombert (2009).

<sup>7</sup>There is vast literature that takes the regulation as given and considers its implications; see, e.g., Merton (1977), Kim and Santomero (1988), Flannery (1989), Furlong and Keeley (1989), Gennotte and Pyle (1991), Rochet (1992), Besanko (1996), and Gorton and Winton (2000). See also Bhattacharya, Boot, and Thakor (1998), Gorton and Winton (2002), and Freixas and Rochet (2008) for good surveys of the literature.

than PTE. The concept of CTE, though not in this name, has been used in the literature on the insurance market with adverse selection; see, e.g., Riley (1975), Rothschild and Stiglitz (1976), Wilson (1977), and Azariadis and Smith (1993). Also, Riley (1975), Rothschild and Stiglitz (1976), and Wilson (1977) show that CTE may not exist.<sup>8</sup>

The rest of the paper is organized as follows. Section 2 gives a simplified version of the model, to illustrate the mechanisms through which the PTE induces overleverage and the CTE rectifies it. Section 3 sets up the full-fledged model. Section 4 examines the PTE and Section 5 the CTE. Section 6 spells the empirical implications of the paper. And Section 7 concludes. Some proofs are relegated to Appendix B, while Appendix A contains detailed examination of the setting without a technical assumption.

## 2 An Example of PTE, Over-Borrowing, and CTE

This example is a simplified version of the model which takes away market segmentation and preference difference, but keeps the feature that the PTE entails over-borrowing, while the CTE yields the social best investment.

In the economy, many entrepreneurs compete for the capital of investors. All economic agents are risk neutral and protected by limited liability. Each entrepreneur has a project and is penniless. The projects have decreasing return to scale and are subject to a common productivity shock: If  $I$  of capital is invested in a project, it will return  $Y = \tilde{A}I^\alpha$ , where  $\alpha < 1$  and  $\tilde{A} = \bar{A}$  with probability  $q$  and  $\tilde{A} = \underline{A}$  with probability  $1 - q$ , with  $0 < \underline{A} < \bar{A}$ . Let  $A_e \equiv q\bar{A} + (1 - q)\underline{A}$  denote the mean.

Assumption E:  $\underline{A} < \alpha A_e$ .

An investor's capital is either invested in projects or in a risk free asset (*storage*) with gross return 1. Investors have abundant capital. Therefore, they get expected return rate 1 in any equilibrium.

---

<sup>8</sup>The existence problem is addressed by Miyazaki (1977), Wilson (1977), Jaynes (1978), and Judd (1984).

The social best amount of investment in each project, denoted by  $I^*$ , solves  $\max_I A_e I^\alpha - I$ . Thus,  $I^* = (A_e \alpha)^{\frac{1}{1-\alpha}}$ .

Move on to competitive equilibrium. Suppose entrepreneurs issue debt to be financed. The face value of \$1 of debt, denoted by  $\bar{R}$ , measures its price.  $\bar{R}$  is really paid in the good state when  $\tilde{A} = \bar{A}$ , and it thus return rate of credit in the state. The bad state return rate of credit  $\underline{R} = \min\{\bar{R}, \underline{A}I^{\alpha-1}\}$ : If the output in that state covers the promised repayment to the creditors, namely if  $\underline{A}I^\alpha \geq I\bar{R}$ , then  $\underline{R} = \bar{R}$ ; otherwise, all the output is distributed to them pro rata and  $\underline{R} = \frac{\underline{A}I^\alpha}{I} = \underline{A}I^{\alpha-1}$ .

### The Price-Taking Equilibrium (PTE)

In the PTE, the representative entrepreneur takes the price of credit,  $\bar{R}$ , as given, and chooses  $I$  to maximize his expected profit,  $q(\bar{A}I^\alpha - \bar{R}I) + (1 - q) \max(\underline{A}I^\alpha - \bar{R}I, 0)$ , in which the term  $\max(\underline{A}I^\alpha - \bar{R}I, 0)$  captures the possibility that he may default in the bad state.

Therefore, a profile of  $(\bar{R}^p, \underline{R}^p, I^p)$  forms a PTE, if

- (i)  $I^p = \arg \max_I q(\bar{A}I^\alpha - \bar{R}^p I) + (1 - q) \max(\underline{A}I^\alpha - \bar{R}^p I, 0)$ ;
- (ii)  $\underline{R}^p = \min\{\bar{R}^p, \underline{A}(I^p)^{\alpha-1}\}$  (the contract is debt);
- (iii)  $q\bar{R}^p + (1 - q)\underline{R}^p = 1$  (the investors get expected return rate 1).

**Proposition E1:**  $I^p > I^*$ , that is, the PTE entails over-borrowing.

**Proof:** First, in the PTE, the debt of entrepreneurs is risky, namely,  $\underline{R}^p < \bar{R}^p$ . Otherwise,  $\underline{R}^p = \bar{R}^p = 1$ ; then by (i),  $I^p = (A_e \alpha)^{\frac{1}{1-\alpha}}$ ; thus  $\underline{A}(I^p)^{\alpha-1} = \frac{\underline{A}}{A_e \alpha} < 1$  by Assumption E; and by (ii),  $\bar{R}^p \leq \underline{A}(I^p)^{\alpha-1} < 1$ , a contradiction.

As entrepreneurs default in the bad state, by (i)  $\bar{R}^p = \bar{A}\alpha(I^p)^{\alpha-1}$  and by (ii)  $\bar{R}^p = \underline{A}(I^p)^{\alpha-1}$ ; and thus  $(q\bar{A}\alpha + (1 - q)\underline{A})(I^p)^{\alpha-1} = 1$  by (iii). It follows that  $I^p = (q\bar{A}\alpha + (1 - q)\underline{A})^{\frac{1}{1-\alpha}} > (A_e \alpha)^{\frac{1}{1-\alpha}} = I^*$ . Q.E.D.

Over-borrowing arises because of the negative externality that the expansion of investment by one entrepreneur decreases  $\underline{R}$ , which has to be compensated in equilibrium with a increased  $\bar{R}$ , namely, a higher price, thus doing harm to all the other entrepreneurs. This effect is not



internalized, because in the PTE, entrepreneurs take the price as given, that is, even if some entrepreneur reduces his investment scale, thus able to provide a higher  $\underline{R}$ , the price for him is not cheaper.

In the CTE below, on the contrary, entrepreneurs obtain cheaper credit by reducing investment scale, and consequently, the social best investment arrives.

### The Contract-Taking Equilibrium (CTE)

In the CTE, entrepreneurs compete with contract  $(\bar{R}, \underline{R}, I)$ : It specifies not only the price ( $\bar{R}$ ) but also the quantity ( $I$ ), which determines  $\underline{R} = \underline{A}I^{\alpha-1}$ .

*A profile of  $(\bar{R}^c, \underline{R}^c, I^c)$  forms a CTE, if*

(i') Given all the other entrepreneurs offer  $(\bar{R}^c, \underline{R}^c, I^c)$ , it is optimal for the representative entrepreneur to offer the same contract;

$$(ii') \underline{R}^c = \underline{A}(I^c)^{\alpha-1};$$

$$(iii') q\bar{R}^c + (1 - q)\underline{R}^c = 1.$$

**Proposition E2:**  $I^c = I^*$ , that is, the CTE yields the social best investment.

**Proof:** Consider the decision problem of the representative entrepreneur, with which (i') is concerned. Let his contract be  $(\bar{R}, \underline{R}, I)$ . Investors are willing to subscribe to it, rather than investing in the risk free asset or other entrepreneurs, if and only if,

$$q\bar{R} + (1 - q)\underline{R} \geq \max(1, q\bar{R}^c + (1 - q)\underline{R}^c) \quad (\text{B})$$

The representative entrepreneur's problem is thus to find  $(\bar{R}, \underline{R}, I)$  to maximize the expected profit  $V = q(\bar{A}I^\alpha - \bar{R}I)$ , subject to (B) above and  $\underline{R} = \underline{A}I^{\alpha-1}$ .

(B) is binding, and together with  $\underline{R} = \underline{A}I^{\alpha-1}$  determines  $\bar{R}$  as an implicit function of  $I$ ;  $\frac{d\bar{R}}{dI} = \frac{d\bar{R}}{d\underline{R}} \cdot \frac{d\underline{R}}{dI} = (-\frac{1-q}{q}) \cdot (\alpha - 1)\underline{A}I^{\alpha-2} = \frac{1-q}{q}(1 - \alpha)\underline{A}I^{\alpha-2}$ . Then,  $\frac{dV}{dI} = q\bar{A}\alpha I^{\alpha-1} - q\bar{R} - I\frac{d\bar{R}}{dI} = q\bar{A}\alpha I^{\alpha-1} - q\bar{R} - qI \cdot \frac{1-q}{q}(1 - \alpha)\underline{A}I^{\alpha-2} = q\bar{A}\alpha I^{\alpha-1} - q\bar{R} - (1 - q)(1 - \alpha)\underline{A}I^{\alpha-1} = q\bar{A}\alpha I^{\alpha-1} + (1 - q)\underline{A}\alpha I^{\alpha-1} - q\bar{R} - (1 - q)\underline{A}I^{\alpha-1} = A_e\alpha I^{\alpha-1} - (q\bar{R} + (1 - q)\underline{R})$ , in the last equation  $\underline{R} = \underline{A}I^{\alpha-1}$  applied.

The right hand side of (B), by (iii'), equals 1. The binding (B) therefore implies that  $q\bar{R} + (1 - q)\underline{R} = 1$ . Then, the solution of  $\frac{dV}{dI} = 0$  is  $I = (A_e \alpha)^{\frac{1}{1-\alpha}} = I^*$ . That is, the representative entrepreneur chooses  $I^c = I^*$  in the CTE. Q.E.D.

Note that the CTE requires commitment power of entrepreneurs:  $I^c < \arg \max_I q(\bar{A}I^\alpha - \bar{R}^c I)$ , that is, given  $\bar{R} = \bar{R}^c$ , they would want to invest more. In fact, it is owing to certain friction such as lacking this commitment power that entrepreneurs cannot obtain cheaper credit by reducing investment scale and the PTE invalidates Welfare Economics Theorem 1. Considering PTE only risks leaving out this kind of friction.

The example is certainly too simple to address the leverage decisions of FIs; indeed, it has no role of financial intermediation, and only presumes debt financing of entrepreneurs. In the model below, investors are split into two groups, differing in risk preference, to endogenize leverage (namely both debt and equity contracts), and in knowledge, to endogenize financial intermediation.

### 3 The Model

In this section, I set up the full fledged model, define two concepts of equilibrium (namely PTE and CTE), and characterize the social best allocation comparable to the equilibrium allocations.

The economy lasts for two dates, today for contracting and investments, tomorrow for return and consumption. It consists of a continuum of households, deep pockets and entrepreneurs. Agents are all protected by limited liability, and within each sector, are identical. The population size of households is  $N$ , and that of deep pockets and of entrepreneurs is both 1. Thus, the quantities chosen by the representative entrepreneur and the representative deep pockets are also the respective aggregate quantities.

The preference of each household is represented by a strictly concave utility function  $U$ , namely,  $U' > 0$  and  $U'' < 0$ . Each household has a small amount of capital to invest, the amount normalized to \$1. Each deep pocket is risk neutral and has \$ $K$ .

The sector of entrepreneurs is as modelled in the preceding section. That is, each entrepreneur is risk neutral as well and has a project, but has no capital of his own. The projects are subject

to a common productivity shock and have decreasing return to scale, possibly due to limited supply of some factors (e.g. land or entrepreneurial human capital), or convex operation cost function. If  $\$I$  is invested in a project, it returns  $Y = \tilde{A}I^\alpha$ , with  $\alpha < 1$ , where  $\tilde{A}$  denotes the common shock. The realization of  $\tilde{A}$  is resolved tomorrow and is the same for all the projects. Today, it is publicly known that  $\tilde{A} = \bar{A}$  with probability  $q$  and  $\tilde{A} = \underline{A}$  with probability  $1 - q$ , with  $0 < \underline{A} < \bar{A}$ . And  $A_e \equiv q\bar{A} + (1 - q)\underline{A}$  denotes the mean.

Capital is either put into 1-to-1 storage, or invested in entrepreneurs' projects.

## The Market Segmentation and Financial Intermediation

Genuine entrepreneurial human capital is observed only by deep pockets, but not by households, and there are many households who always want their fruitless fantasies to be financed. This friction drives deep pockets to become *financial intermediaries* (FIs) that bridge households and entrepreneurs, for the following reasons.

First, given those fantasy-possessed households always want to be financed, whatever pecuniary payoff they can get tomorrow, there are no ways for entrepreneurs to signal their genuine entrepreneurial human capital. As for deep pockets' knowledge of this human capital, there are two other arrangements of utilizing it, besides that of deep pockets becoming FIs.

One, they could simply sell this knowledge to households by recommending to them investable projects, whereby deep pockets become rating agents. This arrangement, however, suffers the problem of rating inflation: A deep pocket may recommend a fake entrepreneur and share with him the funds absorbed.

The other, deep pockets could signal a genuine entrepreneur to households, directly, by investing their own capital in him. This arrangement suffers the same problem as that of rating agency, if the investment of the deep pockets is unobservable or reversible.

However, this problem is not present in the arrangement of deep pockets becoming FIs, which is defined by the feature that deep pockets take the liability to repay households. That is, they have to first repay households before they are permitted to consume any of the investment earnings. If they make "wrong investment", therefore, they undertake loss before the households. By contrast, in the other two alternatives, it is entrepreneurs that take the liability to repay

households (and indeed anyone) who invest in them, and therefore, in any case, these households have no rights to ask deep pockets for repayment; particularly, in the case of the investment failed, deep pockets have no legal duty to make up the households' losses.<sup>9</sup>

Deep pockets, thus, become FIs; hereinafter, "deep pocket" and "FI" are used interchangeably. Similar assumptions of limited participation are made to drive financial intermediation by Adrian and Shin (2008) and He and Krishnamurthy (2008), and to drive arbitrage by Gromb and Vayanos (2002).

Further assumptions are made as follows.

**Assumption 1:** Households cannot split their capital between storage and investment in FIs; neither can FIs between storage and investment in the projects.

Let me first expound the first part of the assumption. Households, in general, would decide how to allocate their capital between storage and investment in FIs. The first part of the assumption simplifies this portfolio choice problem into a bang-bang decision; technically, it substitutes the first order condition ruling the former with the individual rationality constraint ruling the latter. As such, it enables me to focus on the leverage decisions of FIs, and to deliver a clean and clear exposition, in particular, of the mechanism in which leverage boosts the return of equity. Were it absent – the detailed examination of that circumstance is in Appendix A – the exposition would be much more technical, but there would still hold the main results (namely, that the equilibrium leverage is above the social best one in the price-taking equilibrium and that the two coincide in the contract-taking equilibrium).

The second part of Assumption 1 is made, to ensure consistency with the former part, and to give it only the technical significance. If households could freely allocate their capital between storage and investment in FIs, FIs would put all of the capital in hand into the projects and nothing into storage; that is, FIs would face no problems of portfolio choice, because households could choose the portfolios by themselves. Moreover, if FIs were allowed to do portfolio choice,

---

<sup>9</sup>This way of defining financial intermediation by a particular allocation of liability follows Wang (2008) who addresses the friction of costly state verification. Also, in his paper, unlike in this paper, the arrangement of financial intermediation does not always dominate other ones.

the first part of Assumption 1 would become a substantial rather than technical assumption, giving rise to a friction facing households, but not FIs. As such, FIs would acquire an additional role, besides that of bridging households and entrepreneurs, which would complicate the arguments of the paper.

**Assumption 2:**  $\underline{A} < \alpha A_e$  and  $K \leq (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}}$  and  $N > \frac{1}{\underline{A}^{1-\alpha}}$ .

The first inequality of the assumption says that the projects of entrepreneurs poses enough risk, which will imply that they default when  $\tilde{A} = \underline{A}$ , as was the case in the preceding section. The second one says that the capital of deep pockets is scarce. And the third one says that the capital of households is abundant, which will imply that not all of their capital flows to FIs. Each of the three is necessary to drive overleverage in the price-taking equilibrium: If the projects were not so risky, FIs would have no problems with drawing in households' capital; so is true if deep pockets had such abundant capital as to sufficiently reduce the risk passed on to households (as illustrated in the introduction); and if the households' capital were scarce, it would all be drawn into the projects both in equilibrium and in the social best, and no overleverage problem would arise.

There is a capital market on each side of the balance sheet of FIs (namely deep pockets). On the liability side, they compete for households' capital. *Let  $L$  denote the amount of borrowing of the representative deep pocket.* Since risk is costless to deep pockets (who are risk neutral), but costly to households (who are risk averse), competition drive the former to shoulder as much risk for the latter as possible. Therefore, deep pockets offer debt contract to households and contribute their own capital as the equity of FIs, as was illustrated in the introduction. Thanks to Assumption 1, the liability side market is cleared by *a certainty equivalent return rate offered to households, denoted by  $r$ .* That is, if investing in FIs, households obtain  $U(r)$  in equilibrium.  $r \geq 1$ , since households can alternatively choose to store their capital. If  $r = 1$ , they are indifferent between storage and investment in FIs.

On the asset side, entrepreneurs compete for the fund of FIs. *Let  $I$  denote the amount of fund demanded by the representative entrepreneur.* Entrepreneurs are risk neutral; FIs are risk averse, for the sake of their risk averse debt holder; in fact, the profit to deep pockets will be

shown inversely related to the risk on the asset side. Therefore, as deep pockets offer debt to households, so do entrepreneurs to FIs, whereby the investment of FIs is called *credit*. Let  $\bar{R}$  denote the return rate of credit in the good state (when  $\tilde{A} = \bar{A}$ ) and  $\underline{R}$  the rate in the bad state (when  $\tilde{A} = \underline{A}$ ). Then  $\bar{R}$  is the face value (namely the promised amount of repayment) of one dollar of debt and measures the price of credit. As I showed in the preceding section, the seniority of debt claim implies that  $\underline{R} = \min\{\bar{R}, \underline{A}I^{\alpha-1}\}$ : If entrepreneurs do not default in the bad state, that is, the output then covers the promised repayment to the debt, or  $\underline{A}I^{\alpha} \geq \bar{R}I$ , then  $\underline{R} = \bar{R}$ ; if they default, that is,  $\underline{A}I^{\alpha} < \bar{R}I$ , then all the output is distributed to the creditors pro rata, and  $\underline{R} = \frac{\underline{A}I^{\alpha}}{I} = \underline{A}I^{\alpha-1}$ .

There are two possible ways of clearing the credit market. First, it is cleared by the credit price, or equivalently by the credit return rates  $(\bar{R}, \underline{R})$ . Given  $(\bar{R}, \underline{R})$ , entrepreneurs decide the amount of credit to maximize their expected profit. Given  $(\bar{R}, \underline{R})$  on the asset side and  $r$  on the liability side, deep pockets choose the optimal amount they borrow from households; this amount plus the amount of deep pockets' own capital ( $K$ ) gives the aggregate credit supply. The aggregate supply is equalized to the aggregate demand at the equilibrium  $(\bar{R}, \underline{R})$  and  $r$ . Formally, this price-taking equilibrium is defined as follows.

**Definition 1 (PTE)** *A profile of  $\{\bar{R}, \underline{R}, r, I, L\}$  forms a Price-Taking Equilibrium (PTE), if*

- (i)  $\underline{R} = \min\{\bar{R}, \underline{A}I^{\alpha-1}\}$ ;
- (ii)  $I = \arg \max_{I'} \{q(\bar{A}I'^{\alpha} - \bar{R}I') + (1 - q) \max(\underline{A}I'^{\alpha} - \bar{R}I', 0)\}$ ;
- (iii) given  $\{\bar{R}, \underline{R}, r\}$ , each deep pocket chooses the optimal  $L$ ;
- (iv) if  $r > 1$ ,  $L = N$ , and if  $L < N$ ,  $r = 1$ ; and
- (v)  $I = K + L$ .

Condition (i) represents that the contract to FIs is debt, as explained above. Condition (ii) gives the decision problem of the representative entrepreneur; the term  $\max(\underline{A}I'^{\alpha} - \bar{R}I', 0)$  captures the possibility of him default in the bad state. Condition (iii) is self evident. Condition (iv) clears the liability side market: If  $r > 1$ , all households strictly prefer investing in FIs to storage and thus the market is cleared at  $L = N$ ; if only part of households invest in FIs ( $L < N$ ), households must be indifferent between the investment and storage, that is,  $r = 1$ . Lastly,

Condition (v) clears the credit market; note that in the condition  $I$ ,  $K$ , and  $L$  represent the aggregate quantities, as they denote respectively the quantities of the representative entrepreneur or deep pocket.

Second, the credit market might be cleared by contract, *when in the PTE defined above entrepreneurs default in the bad state*, which I will show is the case. In this case, the bad state credit return rate  $\underline{R} = \underline{A}I^{\alpha-1}$ . The good state credit return rate,  $\bar{R}$ , has to be high enough to compensate  $\underline{R}$ , which decreases with  $I$ , the investment scale. If entrepreneurs can lower  $\bar{R}$  charged upon them by committing to a smaller scale of investment ( $I$ ) and hence ensuring a higher  $\underline{R}$ , they do not take the credit price ( $\bar{R}$ ) as given any more, but compete with contract that specifies not only  $\bar{R}$ , but also  $I$  (which determines  $\underline{R} = \underline{A}I^{\alpha-1}$ ). The credit market is then cleared by contract. Given contract  $(\bar{R}, I, \underline{R} = \underline{A}I^{\alpha-1})$  is chosen by all the other entrepreneurs, the representative entrepreneur offers the contract optimal to him. Given the credit return rates offered on the asset side and  $r$  prevailing on the liability side, the representative deep pocket chooses the optimal amount of borrowing. In the equilibrium, the representative entrepreneur's contract coincides with that of all the others, and the credit market clears. Formally, this contract-taking equilibrium is defined as follows.

**Definition 2 (CTE)** *A profit of  $\{(\bar{R}, I, \underline{R}), r, L\}$  forms a Contract-Taking Equilibrium (CTE), if*

(i')  $\underline{R} = \underline{A}I^{\alpha-1}$ ;

(ii') *given all the other entrepreneurs offer contract  $(\bar{R}, I, \underline{R})$ , it is also optimal for the representative entrepreneur to offer the same contract; and*

(iii), (iv) and (v) *of the definition of a PTE above.*

I will examine first the PTE and then the CTE. But before engaged into competitive equilibrium, I shall first have the social best allocation ready, to be compared with the equilibrium allocations.

If all the economic agents were risk neutral, there would be a unique social best allocation which maximizes the total sum of the expected payoff of all the agents. In the economy, deep

pockets and entrepreneurs are risk neutral, but households are risk averse. This preference difference entails a continuum of the social best allocations, depending on the payoff level given to households. By lemma 1, households get  $U(1)$  in both cases of equilibrium. The comparable social best allocation is thus the one that gives them  $U(1)$  also. This allocation is characterized in the following.

### **The Social Best Allocation That Gives Households $U(1)$**

To be consistent with Assumption 1 and comparable with equilibrium, the social planner does not pick a portfolio for households either. That is, he decides the number of the households whose capital is drawn into the projects and their consumption profile, with the consumption profile of all the other households given by storage; rather than he takes in all the capital of the economy, allocates it between storage and investment in the projects, and gives the same consumption profile to all households. The latter is examined in Appendix A, where I analyze in detail the circumstance without Assumption 1 and re-derive the main results of the paper (namely that overleverage occurs in the PTE and that the social best leverage occurs in the CTE).

Suppose the social planner draws  $L$  households' capital into the projects and promise to give each of these households  $\bar{h}$  in the good state and  $\underline{h}$  in the bad state. Then,  $\$K + L$  of capital is invested in each project. After households are paid as promised, deep pockets and entrepreneurs are left to share surplus  $\bar{A}(K + L)^\alpha - L\bar{h}$  in the good state and  $\underline{A}(K + L)^\alpha - L\underline{h}$  in the bad one. The planner's objective is to maximize the expected surplus,  $\max_{L, \bar{h}, \underline{h}} q[\bar{A}(K + L)^\alpha - L\bar{h}] + (1 - q)[\underline{A}(K + L)^\alpha - L\underline{h}]$ , subject to the follow two kinds of constraints.

The first is that the households who contribute capital to the projects get  $U(1)$ , that is,

$$qU(\bar{h}) + (1 - q)U(\underline{h}) = U(1) \tag{1}$$

The second consists of two resource constraints in both states, namely that the social planner has enough output to fulfill his promise to the households:



$$\begin{aligned}\bar{A}(K + L)^\alpha &\geq L\bar{h} \\ \underline{A}(K + L)^\alpha &\geq L\underline{h}\end{aligned}$$

To solve the social planner's problem, I first analyze how the projects' risk should be shared. Risk is costly to the households but costless to deep pockets and entrepreneurs. Therefore, the social planner, whenever possible, wants  $\{\bar{h}, \underline{h}\}$  to be risk free:  $\bar{h} = \underline{h} = 1$ , which is feasible if and only if  $\underline{A}(K + L)^\alpha \geq L$ , or  $L$  is no bigger than the root of  $\underline{A}(K + L)^\alpha = L$ . When it is feasible, the marginal cost of households' capital is 1, while the marginal product is  $M(L) \equiv q\bar{A}\alpha(K + L)^{\alpha-1} + (1 - q)\underline{A}\alpha(K + L)^{\alpha-1}$ .

**Lemma 1**  $M(L) \geq 1$  if  $\underline{A}(K + L)^\alpha \geq L$ .

**Proof.** See Appendix B. ■

The lemma is driven by Assumption 2, which says that both  $K$  and  $\underline{A}$  are small enough.

According to the lemma, the marginal product is above the marginal cost at the maximum level of  $L$  at which the risk free consumption profile is feasible. Therefore, at this maximum level, the planner will not stop drawing households' capital into the project. Hence, in the social best allocation, the consumption profile to the households is risky, namely  $\bar{h} > 1 > \underline{h}$ . To reduce the risk to households as much as possible, the planner give the households all the outputs of the projects in the bad state. That is, the resource constraint in the bad state is binding, which gives:

$$\underline{h} = \frac{\underline{A}(K + L)^\alpha}{L} \quad (2)$$

Then, the social planner's problem becomes:

**Problem 1 (Planner's Problem)**  $\max_{L, \bar{h}, \underline{h}} \bar{A}(K + L)^\alpha - L\bar{h}$ , s.t. (1) and (2).

(1) and (2) define  $\bar{h}$  as an implicit function  $L$ . Then, the derivative of the planner's object above with respect to  $L$  equal to 0 gives:

$$\alpha\bar{A}(K + L)^{\alpha-1} - \bar{h} - \frac{(1 - q)U'(\underline{h})}{qU'(\bar{h})} \frac{\underline{A}(K + L)^{\alpha-1}}{L} (K + (1 - \alpha)L) = 0 \quad (3)$$

The first order conditions of the planner' problem are thus (1), (2), and (3), which altogether characterize the social best  $\{L, \bar{h}, \underline{h}\}$ , denoted by  $\{L^B, \bar{h}^B, \underline{h}^B\}$ .

Finished with the social best allocation, I move on now to competitive equilibrium. In both the PTE and the CTE, the liability side market is cleared in the same way, by certainty equivalent rate  $r$ . In both case,  $r = 1$ , as the households' capital is abundant by Assumption 2:

**Lemma 2**  $r = 1$  in both the PTE and the CTE.

**Proof.** It is implied by Assumption 2 and conditions (iv) and (v) which all hold true in both the PTE and the CTE. If otherwise,  $r > 1$ , then,  $L = N$  by condition (iv) and  $N < I$  by (v) and  $\bar{A}^{\frac{1}{1-\alpha}} < I$  by Assumption 2. On the other hand,  $\frac{\bar{A}I^\alpha}{I} \geq 1$ , that is, as the output of the projects must afford return rate 1 in the good state; otherwise, capital will not be invested in the project, but be stored. It follows that  $I \leq \bar{A}^{\frac{1}{1-\alpha}}$ , a contradiction. ■

So only the credit market needs to be addressed. I am going to examine the PTE in the next section and proceed to the CTE immediately after. In the PTE, FIs are levered above the social best rate, and the problem is rectified by proper capital adequacy regulation.

## 4 The PTE: Overleverage and Regulation

In search of the credit market clearing, I start with the demand side of the credit market, since it is easy.

### 4.1 The Demand Side of the Credit Market

First, I show that entrepreneurs default in the bad state, because the projects are too risky by Assumption 2.

**Lemma 3**  $\bar{R} > 1 > \underline{R}$  in the PTE.

**Proof.** See Appendix B. ■

Therefore, by condition (i):

$$\underline{R} = \underline{A}I^{\alpha-1} \quad (4)$$

Entrepreneurs care for only the profit in the good state, and choose  $I$  to maximize  $\bar{A}I^\alpha - I\bar{R}$ , which implies

$$\bar{R} = \bar{A}\alpha I^{\alpha-1} \quad (5)$$

Basically, (5) decides the demand  $I$ , and then (4) decides the bad state return  $\underline{R}$ .

I move on to the credit supply, which equals  $K + L$ , the sum of FIs' equity capital plus the capital they borrow from households.  $K$  is exogenously given. The decision on  $L$  of the representative deep pocket is examined in the following subsection.

## 4.2 The Supply Side: Risk and Leverage

Frame the situation the representative deep pocket faces. On the asset side, there are two investment channels, one-to-one storage and the risky asset (namely credit to entrepreneurs' projects) of which the gross return rate is  $\bar{R}$  with probability  $q$  and  $\underline{R}$  with probability  $1 - q$ . On the liability side, households are satisfied with  $r = 1$ , that is, in order to take in some households' capital, the deep pocket needs (only) to give payoff level  $U(1)$  to them. Let me temporarily assume that the return rates of the asset satisfy:

$$R_e \equiv q\bar{R} + (1 - q)\underline{R} > 1 \text{ and } qU(\bar{R}) + (1 - q)U(\underline{R}) < U(1) \quad (6)$$

This condition will be proved an equilibrium property later, but in this subsection is treated as an assumption.

The first inequality says that the risky asset has a mean return rate higher than that of storage. The second one says that it is still too risky for households, *even if they could identify the risky asset*.

For the deep pocket, the asset side rate is  $R_e > 1$  and the liability side rate is  $r = 1$ . There seems to be a profitable difference. However,  $r = 1$  is not the expectation rate households are

willing accept, but the certainty equivalent rate; indeed, if they were facing as much risk as the risky asset, by the second inequality of (6), they would demand a expected rate even higher than  $R_e$ . Nevertheless, if the deep pocket can absorb risk for them, as was illustrated in the introduction, he can satisfy them with a lower expected return rate than  $R_e$  and profit from the wedge.

His decision problem is examined in detail as follows.

#### 4.2.1 The Decision Problem of the Deep Pocket

Suppose he takes in  $L$  households' capital, with security that promises to repay each of them  $\bar{h}$  in the good state and  $\underline{h}$  in the bad state. Thanks to Assumption 1, he puts all  $\$K + L$  of capital in hand in the risky asset and does not split it between storage and the risky asset. The investment returns  $(K + L)\bar{R}$  in the good state and  $(K + L)\underline{R}$  in the bad one on the asset side. The total outlay on the liability side is  $L\bar{h}$  and  $L\underline{h}$  respectively. Hence, the deep pocket's expected profit is  $\Pi = q((K + L)\bar{R} - L\bar{h}) + (1 - q)((K + L)\underline{R} - L\underline{h})$ .

His problem is thus to find a profile of  $\{L, \bar{h}, \underline{h}\}$  to maximizes  $\Pi$  subject to the following individual rationality (IR) and limited liability (LL) constraints.

Firstly, the security  $\{\bar{h}, \underline{h}\}$  must guarantee the households with utility level  $U(1)$  and thus satisfy the following IR constraint:

$$qU(\bar{h}) + (1 - q)U(\underline{h}) = U(1)$$

This equation is the same as (1) in the social planner's problem.

Secondly, in both states, the investment must generate enough earning to cover the liability outlay:

$$L\bar{h} \leq (K + L)\bar{R}$$

$$L\underline{h} \leq (K + L)\underline{R}$$

#### 4.2.2 The Characterization of the Optimal Leverage

Let me simplify the problem by considering the difference in rate between the asset side and the liability side of the deep pocket. The expected return rate is  $R_e$  on the asset side and

$h_e \equiv q\bar{h} + (1 - q)\underline{h}$  on the liability side. The deep pocket gains the profit margin  $R_e - h_e$  with each household's capital he draws in. The margin is  $R_e - 1 > 0$ , so long as the security is risk free, namely  $\bar{h} = \underline{h} = 1$ . This is possible if and only if, by the bad state LL constraint,  $L \leq (K + L)\underline{R}$ , equivalently,  $L \leq \frac{R}{1-R}K$ . Therefore, the deep pocket never stops absorbing households' capital if  $L \leq \frac{R}{1-R}K$ . If  $L > \frac{R}{1-R}K$ , the security becomes risky, namely  $\bar{h} > 1 > \underline{h}$ . In order to mostly reduce the risk to the households who invest in him, he let them take all the revenue in the bad state, that is, the bad state LL constraint is binding, which gives:

$$\underline{h} = \frac{(K + L)\underline{R}}{L} \quad (7)$$

And the security to the households is debt, with face value  $\bar{h}$ .

The deep pocket thus has net profit only in the good state. His problem, therefore, becomes:

**Problem 2 (Deep Pocket's Problem:)**  $\max_{L, \underline{h}, \bar{h}} \Pi = q((K + L)\bar{R} - L\bar{h})$  s.t. (1) and (7).

Intuitively, the representative deep pocket profits from trading the risk with the profit margin: He reduces the risk to the households by using his capital as a cushion absorbing the loss to them in the bad state, so that they are willing to accept a lower expected return rate, which gives rise to the profit margin.

Note that the capacity of the deep pocket to absorb the risk for households depends only on  $\underline{R}$ , the bad state return rate, and not on  $\bar{R}$ , the good state return rate: Given  $L$ , the security offered,  $(\bar{h}, \underline{h})$ , is solely determined by  $\underline{R}$ . The lower is  $\underline{R}$ , the less capable is the deep pocket to absorb the risk for the lenders. In particular, if  $\underline{R} = 0$ , he is unable to profit from leverage at all and chooses  $l(\bar{R}, 0) = 0$ : That  $\underline{R} = 0$  implies  $\underline{h} = 0$  for any  $l$  and hence  $\bar{h} > \bar{R}$  by (1) and (6), that is, the margin is negative for any  $l$ .

The first order conditions (FOCs) of the problem consist of (1), (7), and the following:

$$\bar{R} - \bar{h} - \frac{(1 - q)U'(\underline{h})}{qU'(\bar{h})} \frac{KR}{L} = 0 \quad (8)$$

The left hand side (LHS) of (8) equals  $\frac{1}{q} \frac{d\Pi}{dL}$ , considering  $\bar{h}$  as an implicit function of  $L$  through equations (1) and (7), which gives rise to  $\frac{d\bar{h}}{dL} = \frac{d\bar{h}}{d\underline{h}} \cdot \frac{d\underline{h}}{dL} = \left(-\frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}\right) \cdot \left(-\frac{KR}{L^2}\right) = \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} \frac{KR}{L^2}$ .

The simultaneous equations of (1), (7), and (8) decide the optimal  $\{L, \bar{h}, \underline{h}\}$ , given  $\{\bar{R}, \underline{R}\}$ . Notice that all the three equations depend only on  $l \equiv \frac{L}{K}$ , the leverage rate. Thus the optimal leverage scale equals  $L = Kl(\bar{R}, \underline{R})$ , where  $l(\bar{R}, \underline{R})$  is the optimal leverage rate. The expected return rate of the deep pocket's capital,  $\pi$ , is  $\frac{\Pi}{K} = q[(1+l)\bar{R} - l\underline{h}] = q\bar{h} + (1+l)q(\bar{R} - \bar{h})$ . Substitute  $\bar{R} - \bar{h}$  from (8) and note  $\frac{(1+l)KR}{L} = \underline{h}$ ,

$$\pi = q\bar{h} + \frac{U'(\underline{h})}{U'(\bar{h})}(1-q)\underline{h} \equiv \pi(\underline{h}) \quad (9)$$

Through (1),  $\bar{h}$  is an implicit function of  $\underline{h}$ , and  $\frac{d\bar{h}}{d\underline{h}} = -\frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}$ . Thus, the right hand side (RHS) of (9) defines  $\pi$  as a function of  $\underline{h}$ , and  $\frac{d\pi}{d\underline{h}} = (1-q)\underline{h} \cdot d\frac{U'(\underline{h})}{U'(\bar{h})}/d\underline{h} < 0$ : With  $\underline{h}$  increasing,  $\bar{h}$  decreases; then  $U'(\underline{h})$  decreases and  $U'(\bar{h})$  increases; hence  $\frac{U'(\underline{h})}{U'(\bar{h})}$  decreases.

I move on to derive other properties of the optimal leverage rate,  $l(\bar{R}, \underline{R})$ . To intuitively understand them, note that maximizing  $\Pi$  is equivalent to maximizing  $L(\bar{R} - \bar{h})$  and the optimal leverage rate is determined by the trade-off between the leverage scale ( $L$ ) and the profit margin ( $\bar{R} - \bar{h}$ ). On the one hand, given the profit margin, the more the deep pocket borrows, the more he profits. On the other hand, the larger is  $L$ , the smaller is  $\underline{h}$  by (7), the bigger is  $\bar{h}$  by (1), and hence the lower is  $\bar{R} - \bar{h}$ , the margin.

### 4.2.3 The Properties of the Optimal Leverage

First, the debt contract under the optimal leverage bears risk, that is  $\bar{h} > 1 > \underline{h}$ . The debt is risk free if and only if  $l \leq \frac{R}{1-R}$ : Against each unit of investment in the risky asset, the deep pocket can at most borrow  $\underline{R}$  without posing any risk; hence, he has to contribute at least  $1 - \underline{R}$  as the equity of the investment, giving rise to the maximum risk free leverage rate  $\frac{R}{1-R}$ . The optimal leverage rate is beyond, as shown below.

**Proposition 1**  $l(\bar{R}, \underline{R}) > \frac{R}{1-R}$  and hence the debt issued by FIs is risky.

**Proof.** It suffices to show that  $\frac{d\Pi}{dL}|_{L=\frac{R}{1-R}K} > 0$ , so that the deep pocket will keep borrowing at  $l = \frac{R}{1-R}$ .  $\frac{d\Pi}{dL}$  equals  $q$  times the right hand side of (8), which, at  $L = \frac{R}{1-R}K$  and  $\bar{h} = \underline{h} = 1$ , equals  $\bar{R} - 1 - \frac{(1-q)}{q}(1 - \underline{R}) = \frac{1}{q}(R_e - 1) > 0$ . ■

Intuitively, at  $l = \frac{R}{1-\underline{R}}$ , the leverage scale just starts to diminish the profit margin and this negative effect is not in the first order.

**Corollary 1**  $\pi > \frac{R_e - \underline{R}}{1 - \underline{R}}$ .

**Proof.** By simply noticing that  $\frac{R_e - \underline{R}}{1 - \underline{R}} = R_e + \frac{\underline{R}}{1 - \underline{R}} \cdot (R_e - 1)$  gives the return rate of the deep pocket's capital achieved at  $l = \frac{\underline{R}}{1 - \underline{R}}$ , which, by the proposition, is not optimal: Each unit of the deep pocket's capital earns  $R_e$  on its own; based on it,  $\frac{\underline{R}}{1 - \underline{R}}$  units of households' capital can be borrowed at risk free rate, each earning profit margin  $R_e - 1$ . ■

Second, I derive some comparative statics with respect to the return rates on the asset side, and the risk, as is measured by  $\delta = q(1 - q)(\bar{R} - \underline{R})$ . Note that  $l(\bar{R}, \underline{R})$  can be transformed into a function of  $\delta$  and  $R_e$  through  $\bar{R} = R_e + \frac{\delta}{q}$  and  $\underline{R} = R_e - \frac{\delta}{1 - q}$ .

**Proposition 2**  $\frac{\partial l}{\partial \bar{R}} > 0$  and  $\frac{\partial l}{\partial \underline{R}} > 0$ ;  $\frac{\partial l}{\partial R_e} > 0$  and  $\frac{\partial l}{\partial \delta} < 0$ . That is, the optimal leverage rate increases with the returns of the risky asset and decreases with its risk.

**Proof.** See Appendix B. Notice that  $\frac{\partial l}{\partial R_e} > 0$  follows from  $\frac{\partial l}{\partial \bar{R}} > 0$  and  $\frac{\partial l}{\partial \underline{R}} > 0$ . ■

All the comparative statics could be intuitively understood through principle that any variation that expands the margin ( $\bar{R} - \bar{h}$ ) at any given leverage scale ( $L$ ) will give room to increase the scale, hence the leverage rate ( $l$ ).

For  $\frac{\partial l}{\partial \bar{R}} > 0$ : If  $\bar{R}$  increases while  $\underline{R}$  is fixed, then  $\bar{R} - \bar{h}$  is increased for any given  $L$ : Fixed  $\underline{R}$  implies  $\underline{h}$  fixed by (7), hence  $\bar{h}$  fixed by (1), the IR constraint for households.

For  $\frac{\partial l}{\partial \underline{R}} > 0$ : If  $\underline{R}$  increase (with  $\bar{R}$  fixed), then  $\underline{h}$  increases by (7) and hence  $\bar{h}$  decreases by (1), leading to a bigger  $\bar{R} - \bar{h}$ .

Lastly, for  $\frac{\partial l}{\partial \delta} < 0$ : Suppose  $\delta$  increases, with  $R_e$  fixed. Then  $\bar{R}$  increases, but so does  $\bar{h}$ , since  $\underline{h}$  decreases when  $\underline{R}$  decreases. The net effect is negative – hence the scale goes down – as  $\bar{h}$  increases by more than  $\bar{R}$ , for two reasons. One,  $\underline{h}$  decreases by more than  $\underline{R}$  because of leverage:  $d\underline{h} = \frac{1+l}{l}d\underline{R}$  by (7). The other, because the households are risk averse,  $\bar{h}$  has to increase by more than is needed for compensating the decrement in  $\underline{h}$  with the mean fixed:  $d\bar{h} > \frac{1-q}{q}(-d\underline{h})$ . On

the other hand, with  $R_e$  fixed,  $d\bar{R} = \frac{1-q}{q}(-d\underline{R})$ . Therefore,  $d\bar{h} > \frac{1-q}{q}(-d\underline{h}) = \frac{1-q}{q} \cdot \frac{1+l}{l}(-d\underline{R}) = \frac{1+l}{l}d\bar{R} > d\bar{R}$ , and the profit margin decreases.<sup>10</sup>

Previously when I set up the model and defined equilibrium, I intuitively argued that although deep pockets are risk neutral, FIs are risk averse in the sense that the risk on the asset side diminishes the profit of equity (namely of deep pockets). This assertion is proved in the following.

**Proposition 3** *Given  $R_e$ ,  $\frac{d\pi}{d\delta} < 0$ .*

**Proof.** See Appendix B. ■

As  $\frac{d\pi}{d\underline{h}} < 0$ ,  $\frac{d\pi}{d\delta} < 0$  implies that  $\frac{d\underline{h}}{d\delta} > 0$ . Then  $\frac{d\bar{h}}{d\delta} < 0$ , as  $\bar{h}$  is inversely related to  $\underline{h}$ . That is, with  $\delta$  higher, the repayment to the households is smoother. In other words, the bigger the risk on the asset side, counterintuitively, the smaller the risk passed on to the debt holder on the liability side.

Having finished the supply side in this subsection, and the demand side in the preceding one, I move on to the market clearing and the characterization of the PTE.

### 4.3 The Characterization of the PTE

The market is cleared if the demand meets the supply. That is,  $I = K + L = K(1 + l(\bar{R}, \underline{R}))$ . Substitute  $\bar{R}$  and  $\underline{R}$  with (5) and (4) respectively,

$$I = K(1 + l(\bar{A}\alpha I^{\alpha-1}, \underline{A}I^{\alpha-1})) \quad (10)$$

(10) has a unique solution. The left hand side (LHS) of (10) increases with  $I$ , while the right hand side (RHS) decreases with it by Proposition 2. Moreover, if  $I \rightarrow \underline{A}^{\frac{1}{1-\alpha}}$  and thus  $\underline{R} = \underline{A}I^{\alpha-1} \rightarrow 1$ , then  $l \rightarrow \infty$  by Proposition 1; on the other hand, if  $I \rightarrow \infty$  and thus both

---

<sup>10</sup>Adrian and Shin (2008) and Fostel and Geanakoplos (2008) also show that the leverage rate decreases with the risk, but for reasons different from that explained above. The reason in Fostel and Geanakoplos (2008) is that a mean preserving spread lowers  $\underline{R}$  and hence  $\frac{\underline{R}}{1-\underline{R}}$ , the maximum leverage. And the parallel result in Adrian and Shin (2008) is driven by the assumption that a mean preserving spread raises the gain in value of the put option on the riskier asset relative to that on the safer one.



returns go to 0, then  $l \rightarrow 0$ . Therefore, the LHS and RHS intersect, and only once. It follows that *the PTE exists uniquely*.

Substitute (4) into (7),

$$\underline{h} = \frac{A(K+L)^\alpha}{L}$$

This equation is exactly the same as (2) in the social planner's problem.

Substitute (5) and (4) into (8),

$$\alpha \bar{A}(K+L)^{\alpha-1} - \bar{h} - \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} \frac{A(K+L)^{\alpha-1}}{L} K = 0 \quad (11)$$

The simultaneous equations of (1) (the IR to households), (2) and (11) determine  $\{L, \bar{h}, \underline{h}\}$  of the PTE, denoted by  $\{L^P, \bar{h}^P, \underline{h}^P\}$ . Then, the equilibrium leverage rate is  $l^P = \frac{L^P}{K}$ , and the aggregate credit investment is  $I^P = K(1+l^P)$ , which in turn determines the equilibrium returns,  $\bar{R}^P$  and  $\underline{R}^P$ , through (5) and (4) respectively. Both entrepreneurs and deep pockets are paid off only in the good state: An entrepreneur gets  $\bar{A}(I^P)^\alpha - I^P \bar{R}^P = (1-\alpha)\bar{A}(I^P)^\alpha$ , and the return of a deep pocket's capital,  $\pi^P$ , equals, by (9),  $\pi(\underline{h}^P) = q\bar{h}^P + \frac{(1-q)U'(\underline{h}^P)}{U'(\bar{h}^P)} \underline{h}^P$ .

Condition (6) was assumed in Subsection 2.2. Now it is proved a property of the equilibrium.

**Lemma 4**  $q\bar{R}^P + (1-q)\underline{R}^P > 1$  and  $qU(\bar{R}^P) + (1-q)U(\underline{R}^P) < U(1)$ .

**Proof.** See Appendix B. ■

$R_e > 1$  is driven by the scarcity of deep pockets' capital, as was set in Assumption 3. Nevertheless, for any  $K > 0$ , however small it is, leverage will expand the credit supply so much that the credit returns satisfy the inequality  $qU(\bar{R}) + (1-q)U(\underline{R}) < U(1)$ . The intuition is that so long as the returns are so high as to invalidate the inequality, it will be profitable for deep pockets to increase leverage scale, which expands the credit supply, until the credit returns are dragged down to satisfy the inequality.

The lemma points to a caveat associated with the approaches of studying credit shortage that take the credit returns as given, such as those of Holmstrom and Tirole (1997), He and Krishnamurthy (2008), and the partial equilibrium model of Subsection 4.2 of this paper. For

example, the partial equilibrium model shows that if  $\{\bar{R}, \underline{R}\}$  are taken as given, the aggregate credit supply equals  $K + L = K(1 + l(\bar{R}, \underline{R}))$ , linear with  $K$ . It follows that the credit supply goes to 0 when the amount of bank capital goes to 0, which would obviously justify the concern of credit shortage in circumstances when banks suffer substantial capital loss. However, this concern is not obviously justifiable any more, as Lemma 4 above shows, if  $\{\bar{R}, \underline{R}\}$  is endogenized through technologies of decreasing return to investment scale, due to the feedback of credit supply on credit returns: The lower the supply, the higher the returns (under the technologies), and hence the more profitable is it to supply credit.

Note also that Lemma 4 guarantees that the macro model is robust to the concern that households may recognize some genuine entrepreneurs ex post, by, e.g., observing them financed by FIs. Even if that happens, it does not affect financing ex ante: Households will not directly invest in the entrepreneurs even if having recognized them, since  $qU(\bar{R}^P) + (1 - q)U(\underline{R}^P) < U(1)$  by Lemma 4; hence, if households want to invest their dollars in something other than storage, they still have to invest through FIs ex ante.

$\Delta \equiv q(\bar{A} - A_e) = (1 - q)(A_e - \underline{A})$  measures the macroeconomic risk. We have the following comparative statics.

**Lemma 5** *The leverage rate ( $l^P$ ) decreases and the aggregate credit investment ( $I^P$ ) increases with the aggregate level of FIs' capital ( $K$ ). Given  $K$ , the leverage rate and the aggregate credit investment decrease with the macroeconomic risk ( $\Delta$ ).*

**Proof.** It suffices to prove  $\frac{\partial l^P}{\partial K} < 0$ ,  $\frac{\partial I^P}{\partial K} > 0$ , and  $\frac{\partial l^P}{\partial \Delta} < 0$  (with  $A_e$  fixed). See Appendix B. ■

Notice that  $\frac{\partial l^P}{\partial K} < 0$  is an effect of general equilibrium, since by contrast, the leverage rate is independent of  $K$  in the partial equilibrium model of Subsection 4.2. On the other hand, the leverage rate decreases with the risk in general equilibrium as well.

In the next subsection I show that FIs are leveraged over the social best level.

## 4.4 Overleverage

Compare the social best allocation  $\{L^B, \bar{h}^B, \underline{h}^B\}$  with the equilibrium allocation  $\{L^P, \bar{h}^P, \underline{h}^P\}$ . The former is determined by simultaneous equations of (1), (2) and (3), the latter by those of (1), (2), and (11). The difference between (11) and (3) is that the term  $(1 - \alpha)L$  is not present in the former, but is in the latter. This difference leads to the following result.

**Proposition 4**  $L^P > L^B$ , that is, in the equilibrium, FIs are over-leveraged. As a result,  $\underline{h}^P < \underline{h}^B$  and  $\bar{h}^P > \bar{h}^B$ , that is, too much risk is passed on to households.

**Proof.** See Appendix B for the proof of  $L^P > L^B$ . From it,  $\underline{h}^P < \underline{h}^B$  follows through (2). Then  $\bar{h}^P > \bar{h}^B$  follows through (1). ■

As was illustrated in the example of Section 2, the root of overleverage is that entrepreneurs do not take into account the effect that the expansion of investment scale ( $I$ ) pushes up the credit price ( $\bar{R}$ ), and consequently, they over-demand credit, which translates into FIs being overleveraged.

Here is another way of thinking of the overleverage problem, directly related to the decisions of deep pockets. When deciding their individual leverage scales, deep pockets take the aggregate amount of borrowing, and hence  $\underline{R}$ , as given.  $\underline{R}$  decreases with the aggregate amount of borrowing through  $\underline{R} = \underline{A}I^{\alpha-1}$ ; and, as I noted in Subsection 4.2, it solely determines the capacity of each deep pocket to absorb risk for the debt holders. Therefore, overleverage is also due to FIs' failure to take into account the negative externality that the leverage by one FI expands, marginally, the credit supply, and consequently lowers the credit return rate in the bad state, which damages the capacity of other FIs to share risk with their lenders. This externality is reminiscent of the fire sale externality, whereby the fire sale by one agent marginally depresses the price, which tightens the borrowing constraints of other agents and force them to do more fire sale.

**Remark:** This inefficiency is robust to the renegotiation between deep pockets and entrepreneurs. Consider the easy case where an entrepreneur is financed by only one deep pocket who in turn only finances the entrepreneur. Suppose the deep pocket suggests that the entrepreneur invests less,  $I^B = K + L^B < I^P$ , but obtains in the good state  $(1 - \alpha)\bar{A}(I^P)^\alpha$ , exactly what

he would obtain if turning to the credit market. And suppose the entrepreneur accepts the suggestion and signs the agreement, which the deep pocket shows to  $L^B$  households, in order to convince them that they will obtain return  $\underline{h}^B (> \underline{h}^P)$  in the bad state and should thus accept good state return  $\bar{h}^B$  instead of demanding  $\bar{h}^P$ . Were they convinced, the deep pocket would achieve the social best leverage rate, and the PTE would not be robust. However, they will not be convinced, for two reasons. First, the households do not know whether the entrepreneur who signs the agreement is a genuine entrepreneur and thus the agreement is a genuine agreement. Second, even if the agreement is genuine, ex post, the deep pockets' interest is not to honor it but to invest all the  $I^B$  units of capital in the credit market – hence the households are going to obtain bad state return  $\frac{I^B \bar{R}^P}{L^B} < \underline{h}^B$  – due to a debt-overhang problem. The deep pocket is paid off only in the good state; in that state, the repayment to the lenders is the same, but the revenue is  $I^B \bar{R}^P$  by the investment in the credit market and  $\bar{A}(I^B)^\alpha - (1 - \alpha)\bar{A}(I^P)^\alpha$  by honoring the agreement, and  $I^B \bar{R}^P > \bar{A}(I^B)^\alpha - (1 - \alpha)\bar{A}(I^P)^\alpha$ .<sup>11</sup>

In the next subsection I show that the social best leverage is restored in the market by proper capital adequacy regulation.

## 4.5 The Capital Adequacy Regulation

Consider the regulation that restrains a FI from being leveraged over the ratio of  $\frac{L^B}{K}$ , or equivalently, that demands the capital adequacy ratio to be no less than  $\frac{K}{K+L^B}$  (in the book value). Technically, the regulation subjects the deep pocket's problem (Problem 1) to the additional constraint that  $L \leq L^B$ . The effects of the regulation are shown in the following proposition.

**Proposition 5** *Under the regulation that  $l \leq \frac{L^B}{K}$ , the competitive equilibrium implements  $\{L^B, \bar{h}^B, \underline{h}^B\}$ , and the profit of entrepreneurs is decreased, while that of deep pockets increased.*

**Proof.**  $I^P = K + L^P$  is the unique solution of (10) and is larger than  $K + L^B$  by Proposition 4. Therefore,  $K + L^B < K(1 + l(\bar{R}(I^B), \underline{R}(I^B)))$ . It follows that  $L^B < Kl(\bar{R}(I^B), \underline{R}(I^B))$ , that is,  $L^B$

---

<sup>11</sup>The inequality is equivalent to  $(1 - \alpha)\bar{A}(I^P)^\alpha > \bar{A}(I^B)^\alpha - I^B \bar{R}^P$ . As  $I^B < I^P$ , it follows from the fact that  $f(I) \equiv \bar{A}I^\alpha - I\bar{R}^P - (1 - \alpha)\bar{A}(I^P)^\alpha < 0$  for any  $I < I^P$ . That is true, as  $f(I^P) = 0$ , and  $f'(I) = \bar{R}(I) - \bar{R}^P > 0$  for  $I < I^P$ , because  $\bar{R}(I) = \bar{A}\alpha I^{\alpha-1}$  decreases with  $I$  and  $\bar{R}^P = \bar{R}(I^P)$ .

is smaller than the optimal leverage scale of FIs when they face credit returns  $\bar{R}(I^B)$  and  $\underline{R}(I^B)$ . So all FIs would want to borrow more than  $L^B$ , were that not disapproved by the regulation. As a result, the regulation constraint is binding: All FIs are leveraged at  $L = L^B$ . It then follows that  $\underline{h} = \underline{h}^B$  by (2), which in turn implies  $\bar{h} = \bar{h}^B$  through (1). Therefore,  $\{L^B, \bar{h}^B, \underline{h}^B\}$  is implemented.

Consider the effect of the regulation on the payoff of each sector of agents. Households get the same,  $U(1)$ . The entrepreneurs get less: They are paid off only in the good state, with  $\bar{A}I^\alpha - I\bar{R} = (1 - \alpha)\bar{A}I^\alpha$ , and the regulation diminishes  $I$  from  $K + L^P$  to  $K + L^B$ . As a result, deep pockets get more, since the regulation increases the total surplus distributed to entrepreneurs and deep pockets altogether. ■

The fact that the financial sector as a whole gains from the regulation, however, does not mean that individual FIs are happy to abide by it. On the contrary, as the proof above shows, facing the high credit returns induced by this regulation, individual FIs have incentive to be leveraged over the regulation limit. They would like to set up off-balance "Special Investment Vehicles", if they can.

So far I finish examining the PTE. In the equilibrium, entrepreneurs default in the bad state, which gives rise to  $\underline{R} = \underline{A}I^{\alpha-1}$ ; and when deciding the investment scale ( $I$ ), they take the credit price  $\bar{R}$  as given and ignores the effect that the expansion of the investment scale decreases  $\underline{R}$  and consequently increases  $\bar{R}$ . *Suppose that the investment scale is contractible and that entrepreneurs can obtain cheaper credit by committing to a smaller scale.* Under this circumstance, each entrepreneur does not take the credit price prevailing in the market as given, but compete with a contract that specifies the particular price he asks for and the investment scale he commits to. The Contract-Taking Equilibrium (CTE) will arise, which is examined in the following section, and is shown to implement the social best allocation.

## 5 The CTE and the Implementation of the Social Best Allocation

I first characterize the equilibrium allocation, for which, again, I start with the demand side of the credit market

### 5.1 The Demand Side of the Credit Market

In the CTE, the representative entrepreneur takes the contracts offered by all the other entrepreneurs as given and offers the contract optimal to him; in equilibrium, all these contracts coincide.

Suppose that all the other entrepreneurs offer  $(\bar{R}, I, \underline{R} = \underline{A}I^{\alpha-1})$ . Given that, the representative entrepreneur decides what contract  $(\bar{R}', I', \underline{R}' = \underline{A}(I')^{\alpha-1})$  is optimal to him. To solve this problem, the key is to find the combinations of  $(\bar{R}', \underline{R}')$  by which he can attract credit from FIs, given  $(\bar{R}, \underline{R})$  is prevailing on the credit market. The answer is provided below.

**Lemma 6** *Given asset  $(\bar{R}, \underline{R})$  is prevailing, FIs are willing to invest in an asset of which the return rates are  $\bar{R}'$  and  $\underline{R}'$  if and only if*

$$q\bar{R}' + (1 - q)\underline{R}' \frac{U'(\underline{h})}{U'(\bar{h})} \geq q\bar{R} + (1 - q)\underline{R} \frac{U'(\underline{h})}{U'(\bar{h})} \quad (12)$$

where  $(\bar{h}, \underline{h})$  characterizes the debt contract that FIs offer to households when facing  $(\bar{R}, \underline{R})$  on the asset side.

**Proof.** See Appendix B. ■

Indeed, if households are risk neutral, then what matters is the expected return rate and the new asset is acceptable if and only if it gives an expected return rate no smaller than is prevailing on the market, exactly what condition (12) commands, with  $\frac{U'(\underline{h})}{U'(\bar{h})} = 1$ .

The representative entrepreneur's problem is therefore:

$$\begin{aligned} \max_{\bar{R}', I', \underline{R}'} \Gamma &= \bar{A}(I')^\alpha - I'\bar{R}', \text{ s.t. (12), and} \\ \underline{R}' &= \underline{A}(I')^{\alpha-1} \end{aligned} \quad (13)$$

He wants to offer a  $\bar{R}'$  as low as possible. Therefore, constraint (12) is binding and becomes:

$$\bar{R}' + \underline{R}' \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} = \bar{R} + \underline{R} \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}$$

This equation and (13) together determine  $\bar{R}'$  as an implicit function of  $I'$  and  $\frac{d\bar{R}'}{dI'} = \frac{d\bar{R}'}{d\underline{R}'} \frac{d\underline{R}'}{dI'} = (-\frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}) \cdot (\alpha - 1) \underline{A}(I')^{\alpha-2} = (1 - \alpha) \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} \underline{A}(I')^{\alpha-2}$ . Then,  $\frac{dI'}{dI'} = 0$  gives:

$$\bar{A}\alpha(I')^{\alpha-1} - \bar{R}' - (1 - \alpha) \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} \underline{A}(I')^{\alpha-1} = 0 \quad (14)$$

This equation and (13) and (??) compose the FOCs that determine  $(\bar{R}', I', \underline{R}')$  of the representative entrepreneur's problem.

In the equilibrium, the optimal contract of the representative entrepreneur coincides with that of all the others, namely,

$$\bar{R}' = \bar{R}, I' = I, \text{ and } \underline{R}' = \underline{R}$$

Substitute it into (13) and (14), and rearrange:

$$\underline{R} = \underline{A}I^{\alpha-1} \quad (15)$$

$$\bar{R} = \bar{A}\alpha I^{\alpha-1} - (1 - \alpha) \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} \underline{A}I^{\alpha-1} \quad (16)$$

Therefore, in the equilibrium path, the demand side of the credit market, under the CTE, is characterized by (15) and (16), whereas under the PTE, it is characterized by (4) and (5), both in and off the equilibrium path. (15) is the same as (4). But (5) commands that  $\bar{R} = \bar{A}\alpha I^{\alpha-1}$  under the PTE, whereas (16) commands that  $\bar{R} < \bar{A}\alpha I^{\alpha-1}$  under the CTE, namely, the credit price is smaller than the marginal product. Therefore, if  $\bar{R}$  were taken as given, entrepreneurs would want to invest more; to sustain the CTE, it is thus necessary for entrepreneurs to possess the power of committing to a specific investment scale, namely, the scale is contractible.

I move on to the supply side.

## 5.2 The Supply Side

On the credit market, in the equilibrium path,  $(\bar{R}, \underline{R})$  prevails; even off the path, a continuum of entrepreneurs but the representative one offers  $(\bar{R}, \underline{R})$ , and this deviation by a single entrepreneur

should, to keep the flavour of perfect competition, have zero effects upon the market conditions. Therefore, the representative FI's problem is to decide the optimal leverage  $L$ , given  $(\bar{R}, \underline{R})$  prevails on the asset side and  $r = 1$  on the liability side, exactly the same as was his problem in the PTE, examined in Subsection 4.2. The FOCs of the problem are thus (1) and (7) and (8), reproduced and renumbered here:

$$qU(\bar{h}) + (1 - q)U(\underline{h}) = U(1) \quad (17)$$

$$\underline{h} = \frac{(K + L)\underline{R}}{L} \quad (18)$$

$$\bar{R} - \bar{h} - \frac{(1 - q)U'(\underline{h})}{qU'(\bar{h})} \frac{K\underline{R}}{L} = 0 \quad (19)$$

These three simultaneous equations determine  $(L, \bar{h}, \underline{h})$  as functions of  $(\bar{R}, \underline{R})$ .

Now it is time to go to the market clearing, characterize the CTE, and show that it implements the social best allocation.

### 5.3 The Characterization of CTE and the Social Best Allocation

The clearing of the credit market gives:

$$I = K + L$$

Then, (15) becomes  $\underline{R} = \underline{A}(K + L)^{\alpha-1}$ . Substitute into (18),

$$\underline{h} = \frac{\underline{A}(K + L)^\alpha}{L} \quad (20)$$

Substitute (16), (15), and  $I = K + L$  into (19),

$$\alpha\bar{A}(K + L)^{\alpha-1} - \bar{h} - \frac{(1 - q)U'(\underline{h})}{qU'(\bar{h})} \frac{K + (1 - \alpha)L}{L} \underline{A}(K + L)^{\alpha-1} = 0 \quad (21)$$

The simultaneous equations of (17), (20) and (21) determine  $(L, \bar{h}, \underline{h})$  of the CTE, denoted as  $(L^T, \bar{h}^T, \underline{h}^T)$ . The credit investment for each project is then  $I^T = K + L^T$ , which in turn determines the credit returns,  $\bar{R}^T$  and  $\underline{R}^T$ , through (16) and (15) respectively. Both entrepreneurs and deep pockets are paid off only in the good state: An entrepreneur gets  $\bar{A}(I^T)^\alpha - I^T\bar{R}^T = (1 - \alpha)[\bar{A}(I^T)^\alpha + \underline{A}(I^T)^\alpha \frac{(1-q)U'(\underline{h}^T)}{qU'(\bar{h}^T)}]$ , and the return of a deep pocket's capital,  $\pi^T$ , equals, by (9),  $\pi(\underline{h}^T) = q\bar{h}^T + \frac{(1-q)U'(\underline{h}^T)}{U'(\bar{h}^T)}\underline{h}^T$ .



**Proposition 6**  $(L^T, \bar{h}^T, \underline{h}^T) = (L^B, \bar{h}^B, \underline{h}^B)$ , namely, the CTE implements the social best allocation.

**Proof.** Simply by noticing that the equations of (17), (20) and (21) are exactly the same, respectively, as (1), (2) and (3), which altogether determine the social best allocation,  $(L^B, \bar{h}^B, \underline{h}^B)$ .

■

Compare the payoff of each sector of agents between the PTE and the CTE. Households always get  $U(1)$ , the same in both cases.

**Proposition 7** *Deep pockets get less in the CTE than they do in the PTE, and entrepreneurs get more.*

**Proof.** For the first assertion,  $\pi^T < \pi^P \Leftrightarrow \pi(\underline{h}^T) < \pi(\underline{h}^P) \Leftrightarrow \underline{h}^T > \underline{h}^P$ , since  $\frac{d\pi}{d\underline{h}} < 0$  as we saw in Subsection 4.2. In both the CTE and the PTE,  $\underline{h} = \frac{A(K+L)^a}{L}$  decreases with  $L$ . Therefore,  $\underline{h}^T > \underline{h}^P$  follows from  $L^T = L^B < L^P$  by Propositions 6 and 4.

The CTE implements the social best allocation, but the PTE does not. Therefore, the CTE generates a larger surplus for deep pockets and entrepreneurs altogether than the PTE. On the other hand, deep pockets get less in the CTE than they do in the PTE. Hence, entrepreneurs get more in the CTE than in the PTE. ■

This proposition hints a reason for the overleverage of FIs. Even when the investment scale is contractible, banks may collectively decline the entrepreneurs' request for a lower rate backed by the commitment to a smaller investment scale, because the banks would get less otherwise in equilibrium. Particularly, large banks have a higher incentive to decline, since they take into account the effect that approving the request would shrink the credit demand, which may make part of their capital idle.

Parallel to Lemma 5 of the PTE, I find the following comparative statics as to the CTE.

**Lemma 7** *As was in Lemma 5, the leverage rate ( $l^T$ ) decreases and the aggregate credit investment ( $I^T$ ) increases with the aggregate level of FIs' capital ( $K$ ). Given this level, the leverage rate and the aggregate credit investment decreases with the macroeconomic risk ( $\Delta$ ).*

**Proof.** Parallel to the proof of Lemma 5. ■

Again,  $\frac{\partial l^T}{\partial K} < 0$  is an effect of general equilibrium, as was noted as to Lemma 5.

I finish the analysis of the whole model now and proceed to enlist the empirical implications of the paper.

## 6 Empirical Implications

First, on the one hand, Subsection 4.2, which examines the leverage decisions of individual FIs, shows that the leverage of FIs is independent of their capital levels. On the other hand, by Lemmas 5 and 7, in both the PTE and the CTE, the leverage rate decreases with the aggregate level of FIs' capital. Therefore:

**Prediction 1A:** *For a fixed economy during a fixed period, the leverage rate of FIs is independent of the individual levels of their (equity) capital.*

**Prediction 1B:** *Across economies or periods, the average leverage rate decreases with the aggregate level of FIs' capital.*

Some suggestive evidence for Prediction 1A can be found in Adrian and Shin (2007), particularly in their figures 2.3 and 2.4.

Second, by Lemmas 5 and 7,  $\frac{\partial l}{\partial \Delta} < 0$  in both the PTE and the CTE, where  $\Delta$  measures the macroeconomic risk. Therefore:

**Prediction 2:** *The leverage rates of all FIs decrease with the macroeconomic risk.*

**Prediction 2.1:** *If, as Fostel and Geanakoplos (2008) assert, the uncertainty is lower during the boom and higher during the bust, then the leverage rates of FIs are bigger during the boom and smaller during the bust, that is, there is a "leverage cycle" as perceived by Fostel and Geanakoplos (2008).*

Third, by Proposition 4, the PTE induces FIs overleveraged, while by Proposition 6, the CTE yields the socially best leverage. The difference is that in the CTE, entrepreneurs compete

in the dimensions of both price and investment scale, while in the PTE, they compete only in the dimension of price. Therefore, overleverage is due to friction that thwarts entrepreneurs from competing in the dimension of investment scale, e.g. the high auditing costs which render the scale uncontractible. And in the Introduction, I argued that this kind of friction is more likely to present where entrepreneurs are small and dispersed and banks are large and concentrated. Therefore:

**Prediction 3A:** *Across economies, the higher the auditing costs, the more likely are banks to be overleveraged.*

**Prediction 3B:** *Banks are more likely to be overleveraged in economies of which the credit market features small and dispersed firms and large and concentrated banks.*

Fourth, the paper shows that leverage boosts the performance of equity. Since the leverage of commercial banks is restricted by capital adequacy requirements, their equity cannot be fully benefited from leverage, and is thus disadvantaged over the equity of alternative investments (hedge funds, private equities, etc.), if the latter's leverage is unrestricted. This disadvantage might not be fully offset by the advantage of banks enjoying state-provided insurance, which makes the credit to them cheaper than that to alternative investments, when the latter is already cheap enough. In that case, the banks' managers are pressured by shareholders to increase leverage, in order to improve the performance of equity. To summarize:

**Prediction 4:** *If the leverage of alternative investments is not restricted, commercial banks are likely to be (actually) leveraged over the regulatory limits when cheap credit is flooding.*

What happened in the US during the credit boom preceding the recent crisis is an evidence supporting this prediction.

## 7 Conclusion

The paper presents a new approach to examine the leverage decision of financial intermediaries, which, unlike the received approaches, dispenses with borrowing friction, and is based on differing

risk preferences, with equity held by less risk averse agents, debt by those more. This approach captures two real life features: Equity serves to "safe net" debt in rainy days, and debt to boost the return of equity. These two features are intertwined. It is because of being protected by equity that the debt holders are satisfied with a lower expected return than is found on the asset side of the financial intermediaries, and the difference gives rise to the profit margin which the equity earns with debt-financed funds.

With this approach, the paper examines the leverage of financial intermediaries in a segmented economy where they bridge entrepreneurs and households. The efficiency of competitive equilibrium depends on how entrepreneurs compete in the credit market. If they compete only in the dimension of price, namely, if for each entrepreneur the credit price is given and independent of his investment scale, the competitive equilibrium induces financial intermediaries to be overleveraged. If entrepreneurs compete in the dimensions of both price and investment scale, namely, if each entrepreneur asks for a particular price by committing to a specific scale, the competitive equilibrium implements the social best leverage, even under the friction of market segmentation.

Therefore, if leverage is driven by risk sharing, as is modelled in the paper, the friction to be blamed for the overleverage of banks is not market segmentation which drives up financial intermediation, but that owing to which credit demanders (namely entrepreneurs) fail to obtain cheaper credit by committing to a smaller investment scale. Measures that target to overcome this failure will help with the overleverage problem, e.g., those that reduce the auditing costs of small or medium sized firms, and thus make the investment scale more contractible, or those that encourage banks to adapt their rates more closely to the investment scales. To the effect of giving this encouragement, the measure of breaking big banks into small ones might help with the overleverage problem.

# Appendix A: The Analysis of the Model *without* Assumption 1

In this appendix, I analyze what happens with the model if Assumption 1 is absent, that is, if households do the portfolio choice, and so does the social planner. Specifically, I characterize the social best allocations, the PTE, and the CTE in order, whereby I show that the PTE induces overleverage and the CTE implements the social best leverage.

## A1: The Social Best Allocations

As was said, there is a continuum of the social best allocations, depending on the utility level given to households. In the main context, I only considered the one that gives households  $U(1)$ , their equilibrium utility in both the PTE and the CTE. Now with Assumption 1 absent, the equilibrium utility levels might vary, but are both above  $U(1)$  (derived from doing only storage). To be readily comparable with equilibrium allocations, I below characterize all the social best allocations that give households no less than  $U(1)$ .

The social planner's problem is, still, to maximize the surplus for the two risk neutral sectors subject to giving households  $U(e)$ , with  $e \geq 1$ , and to the resource constraints. With Assumption 1 absent, he needs to choose a portfolio consisting of storage and investing in entrepreneurs' projects. That is, today he takes in all the capital,  $K$  units from deep pockets and  $N$  units from households, and stores  $N - L$  units of the capital and invest the rest  $K + L$  units in the projects, with  $\$K + L$  each. Tomorrow, he gives each household  $\bar{c}$  in the good state and  $\underline{c}$  in the bad one, and distribute all the rest to entrepreneurs and deep pockets. His problem is therefore:

$$\begin{aligned} \max_{L, \bar{c}, \underline{c}} q[\bar{A}(K + L)^\alpha + N - L - N\bar{c}] + (1 - q)[\underline{A}(K + L)^\alpha + N - L - N\underline{c}], \text{ s.t.} \\ qU(\bar{c}) + (1 - q)U(\underline{c}) = U(e) \\ \underline{A}(K + L)^\alpha + N - L - N\underline{c} \geq 0 \end{aligned} \tag{22}$$

The second constraint is the resource constraint in the bad state; that in the good state is not binding and thus not included.

As we saw, the planner would want to provide households with the risk free consumption profile,  $\bar{c} = \underline{c} = e$ , which stresses the resource constraint. By Lemma 1, the constraint is binding for the case of  $e = 1$ . Therefore it keeps being binding for the case of  $e > 1$ , in which the constraint is even more stressed. Thus,

$$\underline{c} = \frac{N - L + \underline{A}(K + L)^\alpha}{N} \quad (23)$$

The FOCs for the planner's problem consists of (22), (23) and

$$\bar{A}\alpha(K + L)^{\alpha-1} - 1 + \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})}(\underline{A}\alpha(K + L)^{\alpha-1} - 1) = 0 \quad (24)$$

The system of these three equations determine the social best allocation of  $(L, \bar{c}, \underline{c})$  (in which households get  $U(e)$ ), denoted as  $(L^B, \bar{c}^B, \underline{c}^B)$ .

## A2: The PTE and Overleverage

On the credit market, the demand side is still characterized by (4) and (5). The supply side, again, depends on the leverage level ( $L$ ) of the representative FI. For him, the conditions of the asset side market are still characterized by  $(\bar{R}, \underline{R})$ , but, with Assumption 1 absent, the conditions of the liability side market cannot be simply characterized by  $r = 1$  any more, and become much more complex.

Specifically, the representative FI needs to compete against all other FIs *with a contract* for households' funds. Given all the other FIs offer security  $(\bar{h}, \underline{h})$  to households, the representative FI decides security  $(\bar{h}', \underline{h}')$  optimal for him. For this decision, the key is to find the condition for  $(\bar{h}', \underline{h}')$  to be acceptable to households.

**Lemma A1:** Given  $(\bar{h}, \underline{h})$  prevails on the market for households' funds, a new security  $(\bar{h}', \underline{h}')$  is acceptable to households if and only if

$$\bar{h}' + \underline{h}' \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} \geq \bar{h} + \underline{h} \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} \quad (25)$$

with  $(\bar{c}, \underline{c})$  being the consumption profile of the households who pick the optimal portfolio composed of security  $(\bar{h}, \underline{h})$  and storage. That is,  $(\bar{c}, \underline{c})$  satisfies, with some  $s^* \in [0, 1]$  which denotes

the optimal share of their dollars invested in the security,

$$q(\bar{h} - 1)U'(\bar{c}) + (1 - q)(\underline{h} - 1)U'(\underline{c}) = 0 \quad (26)$$

$$\bar{c} = 1 - s^* + s^*\bar{h} \quad (27)$$

$$\underline{c} = 1 - s^* + s^*\underline{h} \quad (28)$$

**Proof:** Facing securities  $(\bar{h}, \underline{h})$  and  $(\bar{h}', \underline{h}')$ , and also storage, the portfolio choice problem of each household is  $\max_{0 \leq t, s, t+s \leq 1} V(t, s) = qU(1 - t - s + t\bar{h}' + s\bar{h}) + (1 - q)U(1 - t - s + t\underline{h}' + s\underline{h})$ . If  $t = 0$ , that is, if security  $(\bar{h}', \underline{h}')$  is missing, the solution of the problem solve for the simultaneous equations of (26), (28) and (27).  $(\bar{h}', \underline{h}')$  is acceptable if and only if  $\frac{\partial V}{\partial t}|_{t=0, s=s^*} \geq 0 \Leftrightarrow q(\bar{h}' - 1)U'(\bar{c}) + (1 - q)(\underline{h}' - 1)U'(\underline{c}) \geq 0$ , which, through (26), is equivalent to (25). Q.E.D.

As was said, the representative FI will let debt holders have all the revenue in the bad state. His problem is therefore:

$$\begin{aligned} \max_{L', \bar{h}', \underline{h}'} (K + L')\bar{R} - L'\bar{h}', \quad s.t.(25) \text{ and} \\ \bar{h}' = \frac{(K + L')\bar{R}}{L'} \end{aligned} \quad (29)$$

The FOCs for this problem consist of the binding (25), (29), and

$$\bar{R} - \bar{h}' - \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} \frac{K\bar{R}}{L'} = 0 \quad (30)$$

In the equilibrium path, the decision of the representative FI coincides with that of all others:

$$(L', \bar{h}', \underline{h}') = (L, \bar{h}, \underline{h})$$

Thus the portfolio choice problem of households is characterized by the simultaneous equations of (26), (28) and (27); in particular, the share of their dollars invested in FIs is  $s^*$ . Then the clearing of the liability side market gives:

$$L = Ns^* \quad (31)$$

And the clearing of the asset side market gives:

$$I = K + L$$

Substitute (4) (namely  $\underline{R} = \underline{A}I^{\alpha-1}$ ),  $\underline{h}' = \underline{h}$  and  $I = K + L$  into (29),

$$\underline{h} = \frac{\underline{A}(K + L)^{\alpha}}{L} \quad (32)$$

Substitute (5) (namely  $\bar{R} = \bar{A}\alpha I^{\alpha-1}$ ),  $\bar{h}' = \bar{h}$  and  $I = K + L$  into (30),

$$\bar{A}\alpha(K + L)^{\alpha-1} - \bar{h} - \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} \frac{\underline{A}(K + L)^{\alpha-1}}{L} K = 0 \quad (33)$$

The simultaneous equations of (26), (28), (27), (31), (32), and (33) determine  $(s^*, L, \bar{h}, \underline{h}, \bar{c}, \underline{c})$  of the PTE, denoted as  $(s^P, L^P, \bar{h}^P, \underline{h}^P, \bar{c}^P, \underline{c}^P)$ . The equilibrium households' utility is  $qU(\bar{c}^P) + (1 - q)U(\underline{c}^P) \equiv u^P$ .

To be compared to the social best allocation that also gives households  $u^P$ , let the system of these six equations collapse into that of three equations concerning only  $(L, \bar{c}, \underline{c})$  by cancelling  $s^*$ ,  $\bar{h}$  and  $\underline{h}$ . Substitute  $s^* = \frac{L}{N}$  (from (31)) and (32) into (27),

$$\underline{c} = \frac{N - L + \underline{A}(K + L)^{\alpha}}{N}$$

Exactly the same as (23).

To be parallel to the system that determines the social best allocation, let  $\bar{c}$  determined through the following IR constraint:

$$qU(\bar{c}^P) + (1 - q)U(\underline{c}^P) = u^P \quad (34)$$

From (26),  $\bar{h} = 1 + \frac{(1-q)U'(\underline{c})}{qU'(\bar{c})}(1 - \underline{h})$ . Substitute it and (32) into (33) to cancel  $\bar{h}$  and  $\underline{h}$ ,

$$\bar{A}\alpha(K + L)^{\alpha-1} - 1 + \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} (\underline{A}(K + L)^{\alpha-1} - 1) = 0 \quad (35)$$

Therefore,  $(L^P, \bar{c}^P, \underline{c}^P)$  is determined by the system of (23), (34), and (35), while, on the other hand,  $(L^B, \bar{c}^B, \underline{c}^B)$  by that of (23), (22), and (24). With  $U(e) = u^P$ , the two systems differ only between (35) and (24).

**Proposition A1:**  $L^P > L^B$ , namely, the PTE induces overleverage.

**Proof:** Equations (35) and (24) can be uniformly written as  $\bar{A}\alpha(K + L)^{\alpha-1} - 1 + \frac{(1-q)U'(\underline{c})}{qU'(\bar{c})} (\underline{A}\theta(K + L)^{\alpha-1} - 1) = 0$ , with  $\theta = 1$  corresponding to the PTE allocation and  $\theta = \alpha$  to the social best



allocation. This equation, together with (23) and (34), determines  $L$  (and also  $\bar{c}$  and  $\underline{c}$ ) as an implicit function of  $\theta$ . Then  $L^P = L(\theta = 1)$  and  $L^B = L(\theta = \alpha)$ . In a way similar to the proof of Proposition 4, I can prove  $\frac{dL(\theta)}{d\theta} > 0$ . Then  $L^P > L^B$ . Q.E.D.

### A3: The CTE and the Social Best Leverage

Now, with Assumption 1 absent, the condition for the contract  $(\bar{R}', \underline{R}')$  to be acceptable by FIs, given  $(\bar{R}, \underline{R})$  is prevailing on the credit market, is not (12), but the following:

$$q\bar{R}' + (1 - q)\underline{R}' \frac{U'(\underline{c})}{U'(\bar{c})} \geq q\bar{R} + (1 - q)\underline{R} \frac{U'(\underline{c})}{U'(\bar{c})}$$

Through a parallel analysis to that found in the main context, the demand side of the credit market, in the equilibrium path, is now characterized by (15) and

$$\bar{R} = \bar{A}\alpha I^{\alpha-1} - (1 - \alpha) \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} \underline{A} I^{\alpha-1} \quad (36)$$

By contrast, it is characterized by (4) and (5) in the PTE. That is the only change as to the characterization of equilibrium from the PTE to the CTE.

As (15) is the same as (4), we still have (32). Substitute (16) (instead of (5)),  $\bar{h}' = \bar{h}$  and  $I = K + L$  into (30),

$$\bar{A}\alpha(K + L)^{\alpha-1} - \bar{h} - \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} \frac{\underline{A}(K + L)^{\alpha-1}}{L} (K + (1 - \alpha)L) = 0 \quad (37)$$

This equation is the counterpart of (33) of the PTE.

Thus, the simultaneous equations of (26), (28), (27), (31), (32), and (37) determine  $(s^*, L, \bar{h}, \underline{h}, \bar{c}, \underline{c})$  of the CTE, denoted as  $(s^T, L^T, \bar{h}^T, \underline{h}^T, \bar{c}^T, \underline{c}^T)$ . The equilibrium households' utility is  $qU(\bar{c}^T) + (1 - q)U(\underline{c}^T) \equiv u^T$ . In a parallel way, the system of these six equations collapses into the one that determines  $(L^T, \bar{c}^T, \underline{c}^T)$ , consisting of (23) and the following IR constraint

$$qU(\bar{c}^T) + (1 - q)U(\underline{c}^T) = u^T$$

together with

$$\bar{A}\alpha(K + L)^{\alpha-1} - 1 + \frac{(1 - q)U'(\underline{c})}{qU'(\bar{c})} (\underline{A}\alpha(K + L)^{\alpha-1} - 1) = 0$$

These three equations are, respectively, the same as (23), (22) (with  $U(e) = u^T$ ), and (24), which altogether determine  $(L^B, \bar{c}^B, \underline{c}^B)$ . Therefore,

**Proposition A2:**  $(L^T, \bar{c}^T, \underline{c}^T) = (L^B, \bar{c}^B, \underline{c}^B)$ , namely, the CTE implements the social best allocation.

## Appendix B: The Proofs

### The Proof of Lemma 1:

The key condition for this lemma is  $K \leq (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{1}{1-\alpha}}$ . Since  $M(L)$  decreases with  $L$ , it suffices to prove that  $M(L) \geq 1$  for the  $L$  such that  $L = \underline{A}(K + L)^\alpha$ . To see the parallel to the proof of Lemma 3, here let  $\underline{R} \equiv \underline{A}(K + L)^{\alpha-1}$ . Then  $M(L) = \frac{A_e \alpha}{\underline{A}} \underline{R}$ , and  $K + L = (\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}$ . By the latter, the condition  $L = \underline{A}(K + L)^\alpha \Leftrightarrow (\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}} - K = \underline{A}(\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}$ , which is equivalent to

$$(D1): K = f(\underline{R}) \equiv (1 - \underline{R})(\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}.$$

(D1) is parallel to (A1) of the proof of Lemma 3. By Assumption 3,  $K \leq (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{1}{1-\alpha}} = f(\frac{\underline{A}}{\alpha A_e})$ . Then  $\underline{R} = f^{-1}(K) \geq \frac{\underline{A}}{\alpha A_e}$ , as  $f(\underline{R})$  is decreasing. It follows that  $M(L) = \frac{A_e \alpha}{\underline{A}} \underline{R} \geq 1$ . Q.E.D.

### The Proof of Proposition 2:

$l$  is decided by the first order condition  $F(\bar{R}, \underline{R}, l) \equiv \bar{R} - \bar{h} - \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} \frac{\underline{R}}{l} = 0$ , where  $\underline{h}$  and  $\bar{h}$  are functions of  $\underline{R}$  and  $l$  implicitly defined by simultaneous equations of (\*)  $\underline{h} = \frac{(1+l)\underline{R}}{l}$  and (IR)  $qU(\bar{h}) + (1-q)U(\underline{h}) = U(1)$ . Let  $S(l, \underline{R}) \equiv \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}$ . Then,  $\frac{d\bar{h}}{d\underline{h}} = -S$ ,  $\frac{\partial \underline{h}}{\partial \underline{R}} = \frac{1+l}{l}$ ,  $\frac{\partial \underline{h}}{\partial l} = -\frac{\underline{R}}{l^2}$ ,  $\frac{\partial \bar{h}}{\partial \underline{R}} = \frac{d\bar{h}}{d\underline{h}} \frac{\partial \underline{h}}{\partial \underline{R}} = -S \frac{1+l}{l}$ , and  $\frac{\partial \bar{h}}{\partial l} = \frac{d\bar{h}}{d\underline{h}} \frac{\partial \underline{h}}{\partial l} = \frac{S\underline{R}}{l^2}$ . Moreover  $\frac{\partial S}{\partial l} > 0$  and  $\frac{\partial S}{\partial \underline{R}} < 0$ : With  $l$  increasing or  $\underline{R}$  decreasing,  $\underline{h}$  decreases by (\*), then  $\bar{h}$  increases by (IR), then  $U'(\underline{h})$  increases and  $U'(\bar{h})$  decreases since  $U'$  is decreasing, and hence  $S$  increases. Finally, since  $\bar{R} = R_e + \frac{\delta}{q}$  and  $\underline{R} = R_e - \frac{\delta}{1-q}$ ,  $\frac{\partial \bar{R}}{\partial \delta} = \frac{1}{q}$ , and  $\frac{\partial \underline{R}}{\partial \delta} = \frac{-1}{1-q}$ . Then,

$$\frac{\partial F}{\partial l} = -\frac{\partial \bar{h}}{\partial l} - \frac{\partial S}{\partial l} \frac{\underline{R}}{l} + \frac{S\underline{R}}{l^2} = -\frac{S\underline{R}}{l^2} - \frac{\partial S}{\partial l} \frac{\underline{R}}{l} + \frac{S\underline{R}}{l^2} = -\frac{\partial S}{\partial l} \frac{\underline{R}}{l} < 0.$$

$$\frac{\partial F}{\partial \underline{R}} = 1 > 0.$$

$$\frac{\partial F}{\partial \underline{R}} = -\frac{\partial \bar{h}}{\partial \underline{R}} - \frac{\partial S}{\partial \underline{R}} \frac{\underline{R}}{l} - \frac{S}{l} = S \frac{1+l}{l} - \frac{\partial S}{\partial \underline{R}} \frac{\underline{R}}{l} - \frac{S}{l} = S - \frac{\partial S}{\partial \underline{R}} \frac{\underline{R}}{l} > 0.$$

$\frac{\partial F}{\partial \delta} = \frac{\partial F}{\partial \underline{R}} \frac{\partial \bar{R}}{\partial \delta} + \frac{\partial F}{\partial \underline{R}} \frac{\partial \underline{R}}{\partial \delta} = \frac{1}{q} - \frac{1}{1-q} (S - \frac{\partial S}{\partial \underline{R}} \frac{\underline{R}}{l}) = \frac{1}{1-q} (\frac{1-q}{q} - S) + \frac{1}{1-q} \cdot \frac{\partial S}{\partial \underline{R}} \frac{\underline{R}}{l} < \frac{1}{1-q} \cdot \frac{\partial S}{\partial \underline{R}} \frac{\underline{R}}{l} < 0$ , where we apply  $S = \frac{(1-q)U'(h)}{qU'(\bar{h})} > \frac{1-q}{q}$ .

Therefore,  $\frac{\partial l}{\partial \underline{R}} = -\frac{\partial F}{\partial \underline{R}} / \frac{\partial F}{\partial l} > 0$ ,  $\frac{\partial l}{\partial \underline{R}} = -\frac{\partial F}{\partial \underline{R}} / \frac{\partial F}{\partial l} > 0$ , and  $\frac{\partial l}{\partial \delta} = -\frac{\partial F}{\partial \delta} / \frac{\partial F}{\partial l} < 0$ . Q.E.D.

### The Proof of Proposition 3:

Note that  $\pi = \max_l q[(1+l)\bar{R} - l\bar{h}(l, \underline{R})]$ , where  $\bar{h}$  is an implicit function of  $h$  through (1), which in turn is a function of  $l$  and  $\underline{R}$  through (7).  $\frac{\partial \bar{h}}{\partial \underline{R}} = \frac{d\bar{h}}{dh} \frac{\partial h}{\partial \underline{R}} = -S \frac{1+l}{l}$ , where  $S \equiv \frac{(1-q)U'(h)}{qU'(\bar{h})} > \frac{1-q}{q}$  as in the proof of Proposition 2 above. By the Envelop Theorem, given  $R_e$ ,  $\frac{d\pi}{d\delta} = q(1+l) \frac{d\bar{R}}{d\delta} - ql \cdot \frac{\partial \bar{h}}{\partial \underline{R}} \cdot \frac{d\underline{R}}{d\delta} = (1+l) - ql \cdot S \frac{1+l}{l} \cdot \frac{1}{1-q} = \frac{q(1+l)}{1-q} (\frac{1-q}{q} - S) < 0$ . Q.E.D.

### The Proof of Lemma 3:

Otherwise, suppose  $\bar{R} = \underline{R} \equiv R$ . Then  $R = 1$ . Otherwise,  $R > 1$ . With each household's capital, a FI obtains  $R$  with certainty on the asset side, and, by Lemma 1, only needs to pay back 1 to the household, and earns profit  $R - 1$  with certainty. Therefore, all of the households' capital will drawn into the projects, which contradicts Lemma 1.

If  $\bar{R} = \underline{R} = 1$ , consider the credit demand by the representative entrepreneurs, which depends on whether he defaults in the bad state. If he does not, then entrepreneurs choose  $I$  to maximize  $q\bar{A}I^\alpha + (1-q)\underline{A}I^\alpha - I$ , which implies  $I = (A_e \alpha)^{\frac{1}{1-\alpha}}$ . But then the bad state output  $\underline{A}I^\alpha < A_e \alpha I^\alpha = A_e \alpha (A_e \alpha)^{\frac{1}{1-\alpha}} = (A_e \alpha)^{\frac{1}{1-\alpha}} = I$ , which means he defaults, a contradiction. If he defaults in fact in the bad state, he choose  $I$  to maximize  $q(\bar{A}I^\alpha - I)$ , which implies  $I = (\bar{A}\alpha)^{\frac{1}{1-\alpha}} > (A_e \alpha)^{\frac{1}{1-\alpha}}$ . Then by Condition (i) of the definition of the PTE,  $\underline{R} = \underline{A}((\bar{A}\alpha)^{\frac{1}{1-\alpha}}) = \frac{\underline{A}}{\bar{A}\alpha} < \frac{\underline{A}}{A_e \alpha} < 1$ , the last step following Assumption 2, which contradicts the supposition that  $\underline{R} = 1$ . Q.E.D.

### The Proof of Lemma 4:

To prove  $R_e > 1$ , it suffices to show that  $R_e = 1 \Rightarrow K > (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}}$ , which violates Assumption 3. With each unit of borrowed capital, the profit margin of a deep pocket is  $R_e - h_e$ . If  $R_e = 1$ , the margin is 0, as  $h_e \geq 1$ , and is strictly negative when  $h_e > 1$ , which is the case if the repayment  $\{h, \bar{h}\}$  bears any risk by the IR, (1), since households are risk averse. Therefore, if  $R_e = 1$ , leverage happens only if  $\{h, \bar{h}\}$  is risk free, that is,  $h = 1 = \bar{h}$ . Then by the limited liability constraint,  $L \leq (K+L)\underline{R} \Leftrightarrow L \leq \frac{\underline{R}}{1-\underline{R}}K$ . In equilibrium,  $I = K+L$ , therefore,  $I \leq \frac{1}{1-\underline{R}}K \Leftrightarrow K \geq (1-\underline{R})I$ . By (4),  $I = (\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}$ . Therefore,

$$(A1): K \geq (1 - \underline{R})\left(\frac{\underline{A}}{\underline{R}}\right)^{\frac{1}{1-\alpha}}.$$

On the other hand, by (4) and (5),  $\bar{R} = \frac{\bar{A}\alpha}{\underline{A}}\underline{R}$ . So  $q\bar{R} + (1-q)\underline{R} = 1 \Rightarrow \frac{q\bar{A}\alpha + (1-q)\underline{A}}{\underline{A}}\underline{R} = 1 \Rightarrow \underline{R} = \frac{\underline{A}}{q\bar{A}\alpha + (1-q)\underline{A}}$ . By (A1),  $K \geq f\left(\frac{\underline{A}}{q\bar{A}\alpha + (1-q)\underline{A}}\right)$ , where  $f(x) = (1-x)\left(\frac{\underline{A}}{x}\right)^{\frac{1}{1-\alpha}}$ . Since  $f$  is decreasing and  $\frac{\underline{A}}{\alpha A_e} > \frac{\underline{A}}{q\bar{A}\alpha + (1-q)\underline{A}}$ ,  $K \geq f\left(\frac{\underline{A}}{q\bar{A}\alpha + (1-q)\underline{A}}\right) > f\left(\frac{\underline{A}}{\alpha A_e}\right) = (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}}$ , violating Assumption 3. This ends the proof of the former part.

To prove the latter part of the lemma, it suffices to show that if  $qU(\bar{R}) + (1-q)U(\underline{R}) \geq U(1)$ , the optimal  $L = \infty$ , which cannot be true in equilibrium. In order to show that, it suffices to prove  $L = \infty$  for the case where  $qU(\bar{R}) + (1-q)U(\underline{R}) = U(1)$ , because by Lemma 4, higher is  $\tilde{R}$ , the larger is  $L$ .

To prove that, it suffices to show  $\frac{d\Pi}{dL} > 0$  for any finite  $L$ .  $\frac{1}{q}\frac{d\Pi}{dL} = \bar{R} - \bar{h} - \frac{(1-q)U'(\bar{h})}{qU'(\bar{h})}\frac{KR}{L}$ . By (1),  $qU(\bar{h}) + (1-q)U(\underline{h}) = U(1) = qU(\bar{R}) + (1-q)U(\underline{R})$ . It follows that  $q(U(\bar{R}) - U(\bar{h})) = (1-q)(U(\underline{h}) - U(\underline{R})) \Leftrightarrow qU'(\xi)(\bar{R} - \bar{h}) = (1-q)U'(\zeta)(\underline{h} - \underline{R})$ , for some  $\xi, \zeta$  such that  $\bar{R} > \xi > \bar{h} > \underline{h} > \zeta > \underline{R}$ . Then  $\bar{R} - \bar{h} = \frac{(1-q)U'(\zeta)}{qU'(\xi)}(\underline{h} - \underline{R}) = \frac{(1-q)U'(\zeta)}{qU'(\xi)}\frac{KR}{L}$ , where the last equation applies  $\underline{h} = \frac{(K+L)\underline{R}}{L}$  by (7). Substitute this formula for  $\bar{R} - \bar{h}$  into the equation of  $\frac{d\Pi}{dL}$ ,  $\frac{1}{q}\frac{d\Pi}{dL} = \frac{(1-q)KR}{qL}\left(\frac{U'(\zeta)}{U'(\xi)} - \frac{U'(\bar{h})}{U'(\bar{h})}\right)$ . Because  $\underline{h} > \zeta$  and  $U'$  is decreasing,  $U'(\zeta) > U'(\underline{h})$ ; similarly  $U'(\xi) < U'(\bar{h})$  since  $\xi > \bar{h}$ . Therefore,  $\frac{U'(\zeta)}{U'(\xi)} > \frac{U'(\bar{h})}{U'(\bar{h})}$ . Hence  $\frac{d\Pi}{dL} > 0$  for any  $L$ . Q.E.D.

### The Proof of Lemma 5:

First, let us establish that  $\frac{\partial I^P}{\partial K} > 0$ . Suppose otherwise  $\frac{\partial I^P}{\partial K} \leq 0$ . Then,  $\frac{\partial R^P}{\partial K} \geq 0$  and  $\frac{\partial \bar{R}^P}{\partial K} \geq 0$ . By Lemma 4,  $l^P = l(\bar{R}^P, \underline{R}^P)$  increases with  $K$ , which implies  $I^P = K(1 + l^P)$  strictly increases with  $K$ , contradictory to the supposition that  $\frac{\partial I^P}{\partial K} \leq 0$ . Therefore,  $\frac{\partial I^P}{\partial K} > 0$ . Second, that  $\frac{\partial I^P}{\partial K} > 0$  implies that both  $\frac{\partial R^P}{\partial K} < 0$  and  $\frac{\partial \bar{R}^P}{\partial K} < 0$ , which, by Lemma 4, implies  $\frac{\partial l^P}{\partial K} < 0$ .

As to the effects of  $\Delta$ , let us establish  $\frac{\partial I^P}{\partial \Delta} < 0$ , so that  $\frac{\partial l^P}{\partial \Delta} < 0$ , as  $I^P = K(1 + l^P)$  and  $K$  is fixed. Suppose otherwise  $\frac{\partial I^P}{\partial \Delta} \geq 0 \Leftrightarrow \frac{\partial l^P}{\partial \Delta} \geq 0$ . Then,  $\frac{\partial R_e}{\partial \Delta} < 0$ , since  $R_e = ((\alpha q + 1 - q)A_e - (1 - \alpha)\Delta)(I^P)^{\alpha-1}$ . Meanwhile,  $\delta$ , the variance of  $\tilde{R}$ , increases with  $\Delta$ . Both changes strictly decrease  $l^P$  by Lemma 4, contradictory to the supposition that  $\frac{\partial l^P}{\partial \Delta} \geq 0$ . Q.E.D.

### The Proof of Proposition 4:

The only difference is between (11) and (3). These two equations can be unified into

$$(F1): \alpha \bar{A}(K+L)^{\alpha-1} - \bar{h} - \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} \frac{A(K+L)^{\alpha-1}}{L} (K+\theta L) = 0.$$

(F1) is (11) for  $\theta = 0$  (the competitive equilibrium case) and is (3) for  $\theta = 1 - \alpha$  (the social best case). Let  $L(\theta)$  be the solution of (F1), where  $\underline{h}$  and  $\bar{h}$  are implicit functions of  $L$  decided by (2) and (1), which are rewritten below.

$$(F2): \underline{h} = \frac{A(K+L)^\alpha}{L}.$$

$$(F3): qU(\bar{h}) + (1-q)U(\underline{h}) = U(1).$$

Because  $L(0) = L^P$  and  $L(1-\alpha) = L^B$ , to prove  $L^B < L^P$ , it suffices to show that  $\frac{dL}{d\theta} < 0$  for  $\theta \leq 1 - \alpha$ .

Let  $F(L, \theta)$  be the LHS of (F1). Then,  $\frac{dL(\theta)}{d\theta} = -\frac{\partial F/\partial \theta}{\partial F/\partial L}$ . Simply  $\frac{\partial F}{\partial \theta} = -S(L)A(K+L)^{\alpha-1} < 0$ , where  $S(L) = \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})} > 0$ , an implicit functions of  $L$ . To prove  $\frac{dL(\theta)}{d\theta} < 0$ , it suffices to show  $\frac{\partial F}{\partial L} < 0$ .  $\frac{\partial F}{\partial L} = \alpha(\alpha-1)\bar{A}(K+L)^{\alpha-2} - \frac{d\bar{h}}{dL} - \frac{dS}{dL} \frac{A(K+L)^{\alpha-1}(K+\theta L)}{L} - Sd \frac{A(K+L)^{\alpha-1}(K+\theta L)}{L} / dL$ . The first term is negative; as to the third term,  $\frac{dS}{dL} > 0$ , since with  $L$  increasing,  $\underline{h}$  decreases and accordingly  $\bar{h}$  increases, and hence  $S = \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}$  increases. To show  $\frac{\partial F}{\partial L} < 0$ , it thus suffices to prove that  $-\frac{d\bar{h}}{dL} - Sd \frac{A(K+L)^{\alpha-1}(K+\theta L)}{L} / dL < 0$ . By (F3),  $\frac{d\bar{h}}{dL} = -S$ . Then  $\frac{d\bar{h}}{dL} = \frac{d\bar{h}}{d\underline{h}} \frac{d\underline{h}}{dL} = -S \frac{d\underline{h}}{dL}$ . Thus,  $-\frac{d\bar{h}}{dL} - Sd \frac{A(K+L)^{\alpha-1}(K+\theta L)}{L} / dL = S(\frac{d\underline{h}}{dL} - d \frac{A(K+L)^{\alpha-1}(K+\theta L)}{L} / dL)$ , which is negative if and only if  $\frac{d\underline{h}}{dL} < d \frac{A(K+L)^{\alpha-1}(K+\theta L)}{L} / dL$ . Therefore, it suffices to show that  $\frac{d\underline{h}}{dL} < \frac{\partial g}{\partial L}(L, \theta)$  for  $\theta \leq 1 - \alpha$ , where  $g(L, \theta) \equiv \frac{A(K+L)^{\alpha-1}(K+\theta L)}{L}$ .

$\frac{\partial g}{\partial \theta} = A(K+L)^{\alpha-1}$ . Then  $\frac{\partial g \theta}{\partial L} < 0$ . So  $\frac{\partial g L}{\partial \theta} = \frac{\partial g \theta}{\partial L} < 0$ , that is  $\frac{\partial g}{\partial L}(L, \theta)$  decreases with  $\theta$ . Notice that  $\underline{h} = g(L, 1)$ . Then,  $\frac{d\underline{h}}{dL} = \frac{\partial g}{\partial L}(L, 1) < \frac{\partial g}{\partial L}(L, \theta)$  for  $\theta \leq 1 - \alpha$ . Q.E.D.

### The Proof of Lemma 6:

Suppose the representative deep pocket absorbs  $\$L$  from households with security  $(\bar{h}, \underline{h})$  and has thus  $\$K + L$  in hand; and facing assets  $(\bar{R}, \underline{R})$  and  $(\bar{R}', \underline{R}')$ , he invests  $S$  in the former and  $S'$  in the latter. His problem is

$$\max_{S, S', L, \bar{h}, \underline{h}} S\bar{R} + S\bar{R}' - L\bar{h}, s.t.$$

$$qU(\bar{h}) + (1-q)U(\underline{h}) = U(1)$$

$$S + S' = K + L$$

$$\underline{h} = \frac{S\underline{R} + S'\underline{R}'}{L}$$

The second line is his resource constraint and the third line says, as before, that all the bad state revenue go to debt holders. I am looking for the conditions under which  $S' \geq 0$  in the solution of the problem.

The Lagrangian is  $\mathcal{L} = S\bar{R} + S\bar{R}' - L\bar{h} + \lambda(qU(\bar{h}) + (1 - q)U(\underline{h}) - U(1)) + \mu(K + L - S - S') + \gamma(\frac{SR+SR'}{L} - \underline{h})$ . Then,  $\frac{\partial \mathcal{L}}{\partial \bar{h}} = 0 \Rightarrow \lambda = \frac{L}{qU'(\bar{h})}$ , which together with  $\frac{\partial \mathcal{L}}{\partial \underline{h}} = 0$  implies that  $\gamma = \frac{L(1-q)U'(\underline{h})}{qU'(\bar{h})}$ . And  $\frac{\partial \mathcal{L}}{\partial S} = 0 \Rightarrow \mu = \bar{R} + \gamma\frac{R}{L} = \bar{R} + \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}\underline{R}$ .

$S' \geq 0$  in the solution of the problem if and only if  $\frac{\partial \mathcal{L}}{\partial S'}|_{S'=0} = \bar{R}' - \mu + \gamma\frac{R'}{L} \geq 0 \Leftrightarrow \bar{R}' + \gamma\frac{R'}{L} \geq \mu|_{\text{substitute } \gamma \text{ and } \mu} \Leftrightarrow \bar{R}' + \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}\underline{R}' \geq \bar{R} + \frac{(1-q)U'(\underline{h})}{qU'(\bar{h})}\underline{R}$ . Q.E.D.

## References

- [1] Adrian, Tobias and Shin, Hyun Sone (2007) "Liquidity and Leverage," Federal Reserve Bank of New York Staff Reports 328. <http://www.newyorkfed.org/research/staff-reports/sr328.html>
- [2] Adrian, Tobias and Shin, Hyun Sone. "Financial Intermediary Leverage and Value-at-Risk". Federal Reserve Bank of New York Staff Reports, No 338, 2008.
- [3] Arnott, R. J., Greenwald, B. C., and Stiglitz, J. E. (1992), "Information and economic efficiency," *Information Economics and Policy*, 6(1):77-88.
- [4] Bernanke, Ben, Gertler, Mark and Gilchrist, Simon. "The Financial Accelerator and the Flight to Quality." *Review of Economics and Statistics*, 1996, 78(1), pp. 1-15.
- [5] Besanko, David. "The Regulation of Bank Capital: Do Capital Standards Promote Bank Safety?" *Journal of Financial Intermediation*, 1996, 5, pp. 160-183.
- [6] Bhattacharya, Sudipto; Boot, Arnoud W. and Thakor, Anjan V. "The Economics of Bank Regulation." *Journal of Money, Credit, and Banking*, 1998, 30(4), pp. 745-770.
- [7] Boot, Arnoud W., and Anjan Thakor (1993), "Security Design," *Journal of Finance*, 48 (4), 1349-1378.

- [8] Brunnermeier, Markus. "Deciphering the 2007-08. Liquidity and Credit Crunch." *Journal of Economics Perspectives*, 2009, 23(1), pp.77-100.
- [9] Brunnermeier, Markus and Pedersen, Lasse H. 2009. "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, 22(6): 2201-2238.
- [10] Bryant, John. "A Model of Reserves, Bank Runs, and Deposit Insurance." *Journal of Banking and Finance*, 1980, 4, pp. 335 -44.
- [11] Cao, D. (2009), "Collateral Shortages, Asset Price and Investment Volatility with Heterogeneous Beliefs," mimeo, MIT.
- [12] Dewatripont, Mathias and Tirole, Jean. *The Prudential Regulation of Banks*. Cambridge, Massachusetts: The MIT Press, 1994.
- [13] Diamond, Douglas W. and Dybvig, Philip H. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 1983, 91(3), pp. 401–19.
- [14] Diamond, Douglas W. and Rajan, Raghuram G. "A Theory of Bank Capital." *Journal of Finance*, 2000, 55(6), pp. 2431–65.
- [15] Flannery, Mark J. "Capital Regulation and Insured Banks' Choice of Individual Loan Default Risks", *Journal of Monetary Economics*, 1989, 24, pp. 235-258.
- [16] Fostel, Ana and Geanakoplos, John. "Leverage Cycles and the Anxious Economy." *American Economic Review*, 2008, 98(4), pp. 1211-1244.
- [17] Fostel, Ana and Geanakoplos, John (2008), "Collateral Restrictions and Liquidity Under-Supply: A Simple Model," *Economic Theory*, 35, 441-467.
- [18] Freixas, Xavier and Rochet, Jean-Charles. *Microeconomics of Banking*, the 2nd ed. Cambridge, Massachusetts: The MIT Press, 2008.
- [19] Furlong, Frederick T.; Keeley, Michael C. "Capital regulation and bank risk-taking: A note." *Journal of Banking and Finance*, 1989, 13, pp. 883–891.

- [20] Geanakoplos, John (1997), "Promises, Promises." In W.B. Arthur, S. Durlauf and D. Lane (eds.), *The Economy as an Evolving Complex System, II*. Reading MA: Addison-Wesley, pp. 285-320.
- [21] Geanakoplos, John (2003), "Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium," *Advances in Economics and Econometrics: Theory and Applications*, Eighth World Conference, Volume II, Econometric Society Monographs, pp. 170-205.
- [22] Geanakoplos, John and Herakles M. Polemarchakis (1986), "Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete," p.65-95 in W. Heller, R. Starr and D. Starrett (eds.), *Uncertainty, Information and Communication: Essays in Honor of K. J. Arrow*, Vol. III, 65-95, Cambridge University Press.
- [23] Gennotte, Gerard and Pyle, David. "Capital Controls and Bank Risk." *Journal of Banking and Finance*, 1991, 15, pp. 805-824.
- [24] Gorton, Gary. and Pennacchi, George. "Financial Intermediaries and Liquidity Creation." *Journal of Finance*, 1990, 45(1), pp 49-72.
- [25] Gorton, Gary and Winton, Andrew. "Liquidity Provision, Bank Capital and Macroeconomy." 2000, unpublished.
- [26] Gorton, Gary and Winton, Andrew. "Financial Intermediation." NBER Working Papers: No. 8928, 2002.
- [27] Gromb, Denis and Vayanos, Dimitri. "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs," *Journal of Financial Economics*, 2002, 66: 361-407.
- [28] Harris, Milton and Raviv, Arthur. "The Theory of Capital Structure." *Journal of Finance*, 1991, 46(1), pp. 297-355.
- [29] He, Zhiguo and Krishnamurthy, Arvind. "A Model of Capital and Crises.", 2008, NBER working papers: No. 14366.



- [30] Holmstrom, Bengt, Tirole, Jean, "Financial Intermediation, Loanable Funds, and the Real Sector", *Quarterly Journal of Economics*, CXII (1) (1997) 663-691.
- [31] Hombert, Johan (2009), "Optimal Financial Structure and Asset Prices", mimeo, HEC Paris.
- [32] Jensen, Michael C. and William H. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Management*, 1976, 3(4), pp. 305-60.
- [33] Kim, Daesik and Santomero, Anthony M. "Risk in banking and capital regulation." *Journal of Finance*, 1988, 43(5), pp. 1219-1233.
- [34] Kiyotaki, Nobuhiro and Moore, John. "Credit Cycles", *Journal of Political Economy*, 1997, 105(2), pp. 211-248.
- [35] Korinek, Anton (2010), "Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses," mimeo, University of Maryland.
- [36] Liu, Xuewen and Antonio Mello (2008), "The Role of Financial Institutions' Capital Structure in the Liquidity Crisis", mimeo, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1343617](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1343617).
- [37] Lorenzoni, G. (2008), "Inefficient Credit booms," *Review of Economic Studies*, 75, 809-833.
- [38] Merton, R. C. "An analytic derivation of the cost of deposit insurance and loan guarantees: An application of modern option pricing theory." *Journal of Banking and Finance*, 1977, 1(1), pp. 3 – 11.
- [39] Miller, Merton H. "Leverage." *Journal of Finance*, 1991, 46(2), pp. 479-488.
- [40] Modigliani, Franco and Miller, Merton H. "The Cost of Capital, Corporate Finance, and the Theory of Investment." *American Economic Review*, 1958, 48(4), pp. 261-297.
- [41] Morrison, Alan. and White, Lucy. "Crises and Capital Requirements in Banking." *American Economic Review*, 2005, 95(5), pp 1548-1572.

- [42] Morrison, Alan. and White, Lucy. "Is Deposit Insurance a Good Idea, and if so, Who Should Pay for it?" 2006, unpublished, Harvard Business School.
- [43] Myers, Stewart C. "Capital Structure." *Journal of Economic Perspectives*, 2001, 15(2), pp. 81-102.
- [44] Rochet, Jean-Charles. "Capital Requirements and the Behaviour of Commercial Banks." *European Economic Review*, 1992, 36(5), pp. 1137-1170.
- [45] Stiglitz, Joseph, "On the Irrelevance of Corporate Finance Policy", *American Economic Review*, 1974, 64(6): 851-866.
- [46] Townsend, R. (1979), "Optimal Contracts and Competitive Markets with Costly State Auditing", *Journal of Economic Theory*, 21, 265-293.
- [47] Wagner, Wolf (2009), "In the Quest of Systemic Externalities: A Review of the Literature," mimeo, Tilburg University.
- [48] Wang, Tianxi (2008), "The Allocation of Liability, Delegated Monitoring, and Mode of Financing", mimeo, Univ. of Essex.