THE DYNAMICS OF NAMES: A MODEL OF REPUTATION[1]

BY TIANXI WANG[2]

University of Essex, UK

The paper studies reputation of names and its dynamics. Firstly, the paper shows that pure names, backed by nothing intrinsic, can bear reputation. Although the context is purely of adverse selection, a mechanism in the spirit of Kreps (1990) helps sustain reputation. Secondly, it examines how the dynamics of reputation affects the extent of sorting and the level of social efficiency, and derives the dynamics in the equilibria with the highest efficiency. Lastly, it finds a comparative statics result which empirically predicts that brand-names can be fully established sooner in an industry where high-end products have larger profit margins.

1. INTRODUCTION

A name encapsulates past performance and can thus be indicative of the quality of goods and services currently provided under it. However, often past performance is due to people long gone and has nothing to do with current production. How can a name still stand for current quality in these cases? Moreover, reputation is tradeable through mergers and acquisitions or trademark transactions. Unknown firms can acquire reputation by buying "good" names: Tata's acquisition of Jaguar and Lenovo's purchase of the Thinkpad brand from IBM gave them wider recognition. It seems, therefore, that a name's reputation can be backed by nothing intrinsic. On the other hand, it goes up after a success and down after a failure under the name, as if the name stands for some intrinsic type whose posterior is Bayesian updated with the performance. How do we explain this pattern of dynamics if there is actually no such a type?

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Some could still argue that behind names stands something intrinsic, such as Coca Cola's secret recipe. This paper, however, studies whether names backed by nothing intrinsic can bear reputation, and if they can, by which mechanisms. In the paper, the dynamics of the reputation are shaped by social efficiency and vary with economic fundamentals.

Consider an overlapping generation (OLG) economy. Young agents of each generation are of two unobservable types, good or bad, and choose whether or not to produce a widget. They enter production with a name, either forming a new name at no cost or buying an existing name from a retiring agent; the latter carries the history of performances of the previous owners. If names of reputable histories are believed to signal or sort out good types, all newcomers want these names. This belief is rationalized, if good types outbid bad types in the competition for the names. This can happen by the following two mechanisms.

The first is the value-adding mechanism. Consider the case where the type is the only piece of private information. Good type agents succeed in producing a useful widget with a higher probability than bad ones. They outbid bad agents in competing for reputable names if the resale value of the names is higher after a success than after a failure. Therefore, the dynamics of name values decide how well names signal and sort out good types. In this paper, the production by good types generates a social surplus, whereas that by bad types is a social waste. Hence the more sorting, the higher efficiency. The first best, where only good types produce, is not achievable. The second best is implemented by a surprisingly simple dynamics. It involves only *two* states, one represented by names of blank history, the other by names of one period of success, although each history could in principle become a separate state in stationary equilibria and there are an infinite number of histories: the aforementioned two, the history consisting of one failure, of one success after one failure, of two consecutive successes, etc. In the second best, a name is brought into the reputable state by a success and into the non-reputable state by a failure, which explains the "updating" aforementioned.

The second is the commitment mechanism. It comes into being when agents have a second piece of private information, a noisy signal on the quality of their widgets received between the production and the sale of the widgets. This post-production information cannot be transmitted through the names, which were bought before the signals arrive. It is transmitted through the price of widgets if agents are incentivized to price the knowingly useless widgets at the true value. The incentive is driven by the off-equilibrium belief that if the useless widget of a name is overpriced, only bad types will subsequently own that name and they will overprice their widgets. No one then wants the name. This belief essentially subjects names to the *norm of setting honest prices*.[3] An agent who sells a useless widget is thus forced to choose between the name's resale value and the profit from setting a high dishonest price. Buying names with high enough resale values is therefore an ex-ante commitment to pricing widgets honestly, which only good types are willing to make. They are thus sorted out by those names.

In the presence of both pieces of private information (namely both the type and the signal), names of low values are ruled by the value-adding mechanism, and those of high values by the commitment mechanism, with widget prices transmitting the sellers' private information about quality. Moreover, the two mechanisms are complementary. On the one hand, the value-adding mechanism alone cannot implement the second best. On the other hand, only when a name has continuously accumulated enough successes, which push up its value sufficiently high through the value-adding mechanism, can the name bear the commitment effect. Lastly, the second best dynamics now vary with the social surplus generated by good types. The smaller the surplus, the more successes new names have to accumulate before reaching the top reputation. If the surplus is proxied by the profit margin and good types by high-end products, this comparative static result predicts an inverse relationship between the average profit margin of high-end products of an industry and the time span for new brand-names to be fully established in this industry.

1.1. Related Literature. This paper is closely related to the literature on tradeable corporate names in relation to asymmetric information. Kreps (1990) is concerned with moral hazard, Tadelis (1999, 2003), Hakenes and Peitz (2007), and Marvel and Ye (2008) with adverse selection, and Mailath and Samuelson (2001), Tadelis (2002), and Deb (2007) with both. Among the three categories, the present paper belongs to the second one, addressing adverse selection only. A more important difference between these papers concerns the assumption as to the observability of the change of names' ownership. It is unobservable in Tadelis (1999, 2002, 2003) and Mailath and Samuelson (2001), partly observable (to the customers only) in Hakenes and Peitz (2007), and fully observable to

[3] In real life, brand-names are indeed subject to moral judgements. For example, *Financial Times* reported ten most ethically perceived brands in France, Germany, Spain, UK and US respectively (p. 24, Tuesday, Feb., 20, 2007).

the whole market in Kreps (1990), Deb (2007), and the present paper; the latter two actually independently find that names, by bearing reputation, help sorting under full observation of ownership change. Compared to the existing literature, this paper makes three innovations.

Firstly, the paper examines in general how the dynamics of names' reputation, by affecting the extent of sorting, determines the level of social efficiency. Based on this link, the paper derives the dynamics that induce the maximum sorting and the highest efficiency.[4] By contrast, Mailath and Samuelson (2001), Tadelis (2002), and Deb (2007) do not examine this link but consider some dynamics with certain welfare properties (e.g. all the "competent types" choose high effort);[5] Tadelis (1999, 2003) and Hakenes and Peitz (2007), in which all the equilibria are of the same social efficiency, have no efficiency concerns; and Marvel and Ye (2008), while addressing the welfare properties of allowing names to be tradeable, do not compare between equilibria with tradeable names.

Secondly, by introducing the post-production signal, this paper discovers a new mechanism, namely the commitment mechanism, driven by the norm of setting honest prices. This mechanism is in the spirit of Kreps (1990).[6] But note that his paper purely has moral hazard. By contrast, in this paper the commitment mechanism operates in a framework of pure adverse selection. This mechanism is in fact *peculiar* to names' reputation: if reputation were personal rather than based on names, pricing norms would play no role in the absence of moral hazard.[7] Moreover, the commitment mechanism gives a new channel for price to signal quality, which complements the relevant literature in industrial organization (such as Milgrom and Roberts 1986).

Lastly, this paper derives the comparative statics result that predicts an inverse relationship between the time span of building up brand names in an industry and the average profit margin of high-end products of this industry. This is the first time a re-

[4] Liu (2009) derives the dynamics of reputation with costly information acquisition in a framework where a long-live player sequentially plays with short-lived opponents.

[5] In all these three papers production by low types is socially efficient and hence efficiency concerns only moral hazard, whereas in the present paper, production by low types is a social waste and social efficiency is proportional to the extent of sorting.

[6] This mechanism is reminiscent of umbrella branding of Wernerfelt (1988), as it relates a name's pricing behavior of today to that of tomorrow, while umbrella branding relates the quality of one product under the brand-name to that of another.

[7] No norm could stop a bad type mimicking a good type, because his continuation value would depend only on the Bayesian posterior of his type (i.e. his personal reputation), which mimicking helps improve.

sult linking the dynamics of brand names with economic fundamentals is derived in the literature on brand names, including those papers that treat firms as real players with preferences and information (such as Klein and Laffler (1981), Shapiro (1983) and Wernerfelt (1988); see Bar-Isaac and Tadelis (2008) for a good survey of the broad literature on sellers reputation).[8]

In this paper, names bear value in a similar way to fiat money (Samuelson (1958) and Kiyotaki and Wright (1989)). Moreover, a name is essentially a record-keeping device, as is money, according to Kocherlakota (1998). The parallel goes further in equilibria where the value of a name depends only on the number of successes net of failures and not on the order of their occurrences, as if a unit of money were given to the owner because of a success, and were extracted from him because of a failure. Those equilibria, however, are not among the second best in this paper.

The paper consists of two parts, the basic model and the extension. The basic model is intended to highlight the value-adding mechanism and efficiency improvement by names' reputation. The extension delivers the commitment mechanism and the comparative statics, by adding the second piece of private information. Section 2 examines the basic model. Section 3 examines the extension. Section 4 concludes. Some proofs are relegated to the Appendix.

2. THE BASIC MODEL

The basic model is a special case of the extension and is interesting in itself. First we lay out the model.

2.1. The Model. Time goes from $-\infty$ to $+\infty$, with period t starting at date t and ending at date $t+1$. The economy has two goods, corn (endowed good and numeraire) and widget (produced good). Each period is populated with a continuum of sellers of mass two and much more buyers. All agents are risk neutral. Sellers live for two periods, so that in each period mass one sellers are young and the other mass one old. The one-period discount rate for young sellers is $r < 1$. Only young sellers are active. Each of them chooses to produce either one widget at cost c or nothing at all. Old sellers are idle. Sellers consume no widgets but corn. Buyers are endowed with corn and consume

[8] Note that in the present paper, and in the literature on tradeable names in general, the firms are not real players but names consecutively owned by many real players.

both. How long buyers live does not matter since their only role is to compose the long demand side of the widget market in each period. A widget is either useful or useless. A useful widget is worth $\overline{v}$ for the buyer, while a useless one is worth $\underline{v}$. Sellers are of two types, good or bad. A good seller produces a useful widget with probability $\overline{q}$ and a bad one with probability $\underline{q} < \overline{q}$. Without loss of generality, let $\underline{v} = 0$ and $\overline{v} = v$; $\underline{q} = 0$ and $\overline{q} = q < 1$. The proportion of good sellers is γ for each period.

Assumption 1: $\gamma qv < c < qv$.

Since $c < qv$, a good seller generates social surplus $\pi =: qv - c > 0$, whereas a bad seller generates $-c < 0$. Therefore, social efficiency is measured by the extent to which bad sellers are excluded from producing widgets. The question of how to exclude them becomes interesting because of the following information structure.

A seller's type, good or bad, is his private information. The quality of a widget, useful or useless, is not observable to the buyer when it is traded, but is revealed to all the agents of this and the next generations at the end of the period by word of mouth.

Assumption 2: Although the quality becomes publicly known at the end of the period, it is not contractible when the widget is traded.

This assumption implies that the price of a widget cannot be based on its quality. Otherwise, if it is priced at its value, v or 0, bad sellers will never enter production and the question of how to exclude them becomes trivial.[9]

Suppose that after knowing his type but before engaging in production, a young seller obtains a name for his firm. He either forms a new name at no cost or buys an existing name from a retiring seller. Then the trading of names becomes the only inter-period link of the economy. All unsold names retire out of the economy with the owners. As only young sellers hold names, it is common knowledge that ownership of names changes each period.

[9] The same assumption is made by Tadelis (1999, 2002, 2003). This assumption is in the spirit of the Holmstrom (1999) career concern model, where at the end of a period a manager's performance is perfectly observed by the labor market, but at the beginning his wage contract cannot be based on it. The fundamental difference is that in Holmstrom (1999), the manager never dies and hence it is about personal reputation, whereas in this paper each seller retires after one period and it is about organizational reputation.

Sellers of each generation go through the following stages in order.

1. Young sellers are born, privately know their types, and decide whether to produce a widget.

2. The name market opens, where young sellers buy names from retiring sellers of the last generation.

3. Widgets are produced, and sold in the widget market to buyers, who do not observe the quality of the widgets

4. This period ends, the next starts, and the new generation of young sellers are born. The quality of all the widgets is publicly revealed to them (and the future buyers).

5. The name market opens again where names are traded between the retiring sellers of this generation and the newborn young sellers.

If a seller buys his name at price p_0, sells his widget at w, and resells the name at p_1, then his overall return is $R = -p_0 + w - c + rp_1$, in which $w - c$ is the profit from the widget market and $-p_0 + rp_1$ is the capital gain from the name markets. The utility of a widget buyer is $\widetilde{v} - w$, where $\widetilde{v} = v$ or 0, depending on its quality. The reservation value of sellers who do not produce and of buyers who do not purchase are both 0.

A name could be used consecutively over several periods. The history of a name until date t is defined as the sequence of the qualities of the widgets produced up to period $t-1$ under that name. This history is publicly known in period t. A name is characterized by its history; trading names is essentially trading histories. Let "s" denote success (a useful widget is produced), "f" failure (a useless widget is produced) and "h" a history. A name with history h is called an h-name. h is either empty (for new names), denoted by "ϕ", or a sequence consisting of s and f, such as "s", sf", "$sssf$" etc. Let "s^n" be the abbreviation of n consecutive "s", and similarly for f^n. Denote by H^n the set of all histories of length n. Then, $H^0 = \{\phi\}$, $H^1 = \{s, f\}$, and $H^2 = \{s^2, sf, fs, f^2\}$ etc. And let $H = \bigcup_{n \geq 0} H^n = \{\phi, s, f, s^2, sf, fs, f^2, ...\}$ be the set of all possible histories.

Names evolve with the performance of their owners. If an h-name is owned by a good seller, then with probability q, he succeeds, which transforms it into an hs-name in the next period, while with probability $1 - q$ the failure transforms it into an hf-name. If an h-name is owned by a bad seller, it will definitely become an hf-name.

The equilibrium concept is competitive equilibrium, which consists of prices and decisions. The prices of widgets are denoted by w_{ht} and the prices of names by p_{ht}, where subscript h represents the histories of names and t the dates of trading. Only young sellers have decisions to make. They first decide whether to produce and then which names to buy. Let $e_{Bt}, e_{Gt} \in [0,1]$ denote the probability of a bad seller and a good seller entering production respectively and let λ_{ht} denote the proportion of good sellers among the owners of all h-names in period t. Let ρ_{ht} denote the mass of h-names in the date t name market. The total value of the names in the market, which is the transfer from generation t sellers to generation $t-1$, is $V_t = \sum_{h \in H} \rho_{ht} p_{ht}$.

Definition 1 $\{p_{ht}, w_{ht}; e_{Bt}, e_{Gt}, \lambda_{ht}\}_{h,t}$ *constitutes a competitive equilibrium if and only if*

(i): Given the prices $\{p_{ht}, w_{ht}\}_{h,t}$, *the optimal decisions of sellers at date t are summarized by* e_{Bt}, e_{Gt}, *and* λ_{ht}.

(ii): Given the decisions $\{e_{Bt}, e_{Gt}, \lambda_{ht}\}_{h,t}$, p_{ht} *clears the market of h-names at date t.*

(iii): Given the decisions $\{e_{Bt}, e_{Gt}, \lambda_{ht}\}_{h,t}$, w_{ht} *clears the market of the widgets of h-names at date t.*

(iv) (No Ponzi): $\lim_{t\to\infty} r^t V_{T+t} = 0$ *for any T.*

All the conditions but no Ponzi are self evident. For the no Ponzi condition, let R_t be the total return of generation t sellers and Π_t their total profit from the widget market. Besides the profits, they pays V_t in total to buy names and obtain V_{t+1} in total from selling the names. Therefore, $R_t = \Pi_t - V_t + rV_{t+1}$. Then, if and only if the no Ponzi condition holds, we have

$$\sum_{t\geq 0} r^t R_{T+t} = \sum_{t \geq 0} r^t \Pi_{T+t} - V_T \tag{1}$$

The no Ponzi condition ensures that, taking as a whole all the sellers from generation T onwards, their return must come from the "real values" they create in producing widgets. Thus sellers cannot earn arbitrarily large resources by simply buying and reselling names. The no Ponzi condition is used here to prick asset bubbles, as in macroeconomics literature.

Only "*stationary equilibria*" are considered in this paper, where $p_{ht}, w_{ht}, e_{Bt}, e_{Gt}$, and λ_{ht} do not depend on t, but on h only. In stationary equilibria, names are classified into states. Names in the same state have the same value and evolve into the same state after a success or a failure. The dynamics of names are then Markovian transformations over the states. In principle, each history could be a separate state, and there are an infinite number of histories. What happens in equilibrium, however, is much simpler, as will be shown. States are denoted by capitalized histories, such as Φ, S, and S^2, which respectively denote the states containing new names, s-names, and s^2-names.

Since new names are created at no cost, $p_\phi = 0$. Since buyers are on the long side of the widget market, competition drives them to obtain their reservation value, 0, in any equilibrium. This, in combination with them being risk neutral, implies that the market clearing price of a widget equals the expected value:

$$w_h = E(v|h) = q\lambda_h v \tag{2}$$

Lemma 1 *In any equilibrium, bad sellers obtain 0 return.*

Proof. Suppose otherwise, in some equilibrium bad sellers get positive return from production. Then they all enter production in the equilibrium. The average price of the widgets, equal the expected value, is thus γqv, while the average cost is c. By Assumption 1, $\gamma qv < c$. The no Ponzi condition therefore implies that on average sellers obtain less than their reservation value, which is impossible in any equilibrium. ■

As both buyers and bad sellers get 0 surplus, all social surplus goes to good sellers. Their return, therefore, measures the social efficiency.

Before we show how names bear reputation, we first consider as a benchmark what happens if names do not bear reputation, which sheds lights on why personal reputation is not sufficient and why we need names' reputation.

2.2. *Benchmark: the Babble Equilibria.* Suppose names are not believed to convey any information about the type of the current sellers. Then young sellers are not willing to pay for existing names, and the following proposition holds.

Proposition 1 *In any equilibrium where names do not bear reputation, the social surplus is 0.*

Proof. In the equilibria where no widgets are produced, the social surplus is obviously 0. For the equilibria where widgets are produced and traded, first note that the following profile of prices and decisions forms an equilibrium. $p_h = 0$ and $w_h = c$ for all h; no sellers buy existing names (as discussed above), and the two types of sellers enter production in such a proportion that the expected value of the widgets is c. Given the prices, the return of both types of sellers is $w - c = 0$. Thus they are indifferent between entry and not, and any proportion of entry is justifiable. Given the entry decisions, the price of widgets is c by (2). The price of all names is surely 0. Thus this is an equilibrium.

No other prices are possible in the equilibria. Names' price has to be 0. If the price of widgets $w < c$, no sellers want to produce. If $w > c$, then all sellers, particularly the bad ones, get positive return if entering production, which contradicts Lemma 1. Thus, $w = c$ in all the equilibria. Hence, the return of good sellers is always 0 and so is the social surplus. ■

In this benchmark, bad sellers enter production to the extent that all the social surplus generated by good sellers is wholly dissipated. At the end of each period, a mass q of retiring good sellers has succeeded and established personal reputations. However, in the next period these people are retired and bring their personal reputations out of the economy. This explains why such inefficiency arises in the benchmark. Having names bear reputation improves efficiency, exactly because names can technically live forever, but persons cannot.

The next subsection provides a complete characterization of equilibrium payoffs and thus shows what can be done by having names bear reputation.

2.3. *The Characterization of Equilibrium Efficiency.* A series of equilibria, ordered by $\lambda \in [\frac{c}{qv}, 1]$, are constructed below. The efficiency of the equilibria increases with λ; when $\lambda = 1$, we arrive at the highest efficiency, and when $\lambda = \frac{c}{qv}$, we go back to the babble equilibria, the lowest efficiency.

Two-State $\lambda-$Equilibrium (TSE-λ). In this equilibrium, names are of two states, Φ and S, for new names and reputable names respectively. A Φ-name becomes an S-name after a success and remains a Φ-name after a failure. An S-name remains an S-name after a success, and degenerates into (or is replaced by) a Φ-name after a failure. The dynamic is illustrated as follows.

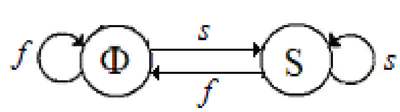

Figure 1

The Two-State Dynamics

The prices of names are $p_\Phi = 0$ and $p_S = \lambda qv - c$. The prices of widgets are $w_S = \lambda qv$ and $w_\Phi = c$. The decisions are summarized by $\lambda_S = \lambda$ and $\lambda_\Phi = \frac{c}{qv}$; $e_G = 1$ and $e_B = \frac{\gamma}{1-\gamma} \frac{\lambda(1-q)(1-\lambda_\Phi)+(1-\lambda)(q\lambda_\Phi+1-\lambda_\Phi)}{\lambda_\Phi}$.[10]

Lemma 2 *The above profile of prices and decisions forms an equilibrium for any* $\lambda \in [\frac{c}{qv}, 1]$.

Proof. Let us verify that conditions (i)-(iv) are satisfied. Obviously, condition (iii), equivalent to (2), is satisfied, and so is the no Ponzi (condition (iv)).

For condition (i), check that both types of sellers are indifferent in buying either state of names, and that good sellers get nonnegative return while bad sellers get 0. Hence any λ_S, λ_Φ and e_B are optimal, and the optimal $e_G = 1$. If good sellers buy S-names, their return is $-p_S + w_S - c + r[qp_S + (1-q)p_\Phi] = rqp_S$. If they buy Φ-names, the return is $-p_\Phi + w_\Phi - c + r[qp_S + (1-q)p_\Phi] = rqp_S$ again, where $p_S = \lambda qv - c \geq 0$. Hence good sellers are indifferent between buying Φ- and S-names and prefer entering production. If bad sellers buy S-names, the return is $-p_S + w_S - c + rp_\Phi = 0$. If they buy Φ-names, it is $-p_\Phi + w_\Phi - c + rp_\Phi = 0$. Hence, bad sellers are indifferent between buying any names, and between entry and not.

For condition (ii), given that good sellers buy both states of names on the equilibrium path, they must be indifferent between buying either state of names at the market clearing price of S-names. That is, $-p_S + w_S - c + rqp_S = w_\Phi - c + rqp_S \Rightarrow p_S = w_S - w_\Phi = \lambda qv - c$, as specified above. ■

In TSE-λ, the return of good sellers, which measures efficiency, is $rq(\lambda qv - c)$. It increases continuously with λ. Measured by the number of the states of names, the TSE are the simplest after the babble equilibria; the former involves two states, Φ and S, while the latter involves only one. By Proposition 1, the latter implements the lowest social efficiency. Therefore, it is surprising that, with only one more state added, the TSE already implement all levels of equilibrium efficiency with $\lambda \in [\frac{c}{qv}, 1]$. The following lemma helps prove this assertion.

[10] To find e_B, first notice that in the steady state, the inflow of state S equals the outflow, that is, $\rho_\Phi \lambda_\Phi q = \rho_S[\lambda(1-q)+1-\lambda]$. As all good sellers enter ($e_G = 1$), and they own either Φ-names or S-names, $\gamma = \rho_\Phi \lambda_\Phi + \rho_s \lambda$. From these two equations, we find ρ_Φ and ρ_S. Then the total mass of bad sellers entering production is $\rho_\Phi(1 - \lambda_\Phi) + \rho_s(1-\lambda)$, which divided by $1-\gamma$ gives e_B.

Lemma 3 *$p_h \le \frac{\pi}{1-r}$ for any h in equilibrium.*

Proof. See Appendix C. ■

Intuitively, if some h-names are sold at a price higher than $\frac{\pi}{1-r}$, for sellers to buy these names, the sum of the names' resale values $(q\lambda_h p_{hs} + (1-q\lambda_h)p_{hf})$ must be even higher. The same consideration holds true for those names that evolve from the h-names (namely, hs, hf, hs^2, hsf etc.), which pushes the sum total of the values of these subsequently evolved names higher and higher, and in the end breaks the no Ponzi condition.

The main proposition of the basic model is stated here.

Proposition 2 *The surplus of TSE-1, $rq\pi$, is the maximum social surplus among all equilibria. Therefore, the series of TSE-λ implement all levels of equilibrium efficiency with $\lambda \in [\frac{c}{qv}, 1]$.*

Proof. Given any equilibrium, we are going to show that the equilibrium return of good sellers is not greater than $rq\pi$. For the equilibrium, $P = \sup\{p_h | h \in H\}$ is well defined by lemma 3. For any ε such that $0 < \varepsilon < c$, there exist h^*-names such that $p_{h^*} > P - \varepsilon$. First, not all h^*-names are bought by bad sellers in equilibrium. Otherwise, the names sort out useless widgets, and $w_{h^*} = 0$. Buying the names, bad sellers obtain $-p_{h^*} + w_{h^*} - c + rp_{h^*f} \le -p_{h^*} - c + rP < -P + \epsilon - c + rP < 0$. Thus they should not buy the names, a contradiction.

Thus h^*-names are bought by good sellers on the equilibrium path. The return of good sellers buying the names is $-p_{h^*}+w_{h^*}-c+r[qp_{h^*s}+(1-q)p_{h^*f}] \le -p_{h^*}+w_{h^*}-c+r[qP+(1-q)p_{h^*f}] = rq(w_{h^*}-c)+rq(P-p_{h^*})+(1-q)(-p_{h^*}+w_{h^*}-c+rp_{h^*f})+q(1-r)(-p_{h^*}+w_{h^*}-c)$. Let us check the last sum term by term. For the first two terms, $w_{h^*} - c = \lambda_{h^*}qv - c \le \pi$ by (2), and $P - p_{h^*} < \varepsilon$. As to the third and the fourth terms, consider what bad sellers get if they buy h^*-names. Their return is $-p_{h^*} + w_{h^*} - c + rp_{h^*f}$, which is nonpositive. It follows that the fourth term $-p_{h^*} + w_{h^*} - c \le -rp_{h^*f} \le 0$. Therefore, the return of good sellers buying h^*-names is no bigger than $rq\pi + rq\varepsilon$. This return is the equilibrium return of good sellers, since they buy h^*-names on the equilibrium path. The equilibrium return is thus no bigger than $rq\pi + rq\varepsilon$, for any ε such that $0 < \varepsilon < c$. The proposition is proved by making ε go to 0. ■

For intuition, consider the extreme case where $r = 1$ and some h^*-names actually take the top value P. The return of good sellers buying these names consists of the profit,

$w_{h^*} - c$, and the capital gain. As the h^*-names take the top value, the sellers obtain no capital gain from success. However, when they fail, the capital loss must be no less than the profit. Otherwise, bad sellers earn positive return by buying the h^*-names. Therefore, at least $1 - q$ of the profits are offset by the expected capital loss and the return is thus no more than $q(w_{h^*} - c) = q(\lambda_{h^*} qv - c) \leq q(qv - c) = q\pi$.

By Proposition 2, we know the first best is not achievable: in the first best, only good sellers produce and hence the social surplus is $\pi > rq\pi$. Therefore names function as only an imperfect substitute for the contracts that would implement the first best when the quality of widgets is verifiable.

There are equilibrium dynamics that involve more than two states. An example is given in Appendix A, which involves four states. It implements the second best efficiency, however, if and only if it degenerates to the two-state dynamics above, that is, among the four states, two of them are equivalent and so are the other two.

Here we end the examination of the basic model. The next section extends it by adding one element, the post production signal. The extension delivers two points that the basic model fails to deliver. It shows that even in a framework of purely adverse selection, a norm of setting honest prices helps sustain names' reputation. Also, it shows that in the second best equilibria, the smaller is π, the greater is the number of successes a name needs to accumulate in order to accomplish the top reputation, whereas in the basic model, only one success is needed to accomplish the top reputation, independent of π.

3. THE EXTENSION

Subsection 3.1 first presents the new element, the post-production signal. To utilize this extra information, the norm of setting honest prices for widgets is introduced. This norm drives a new mechanism to sort out good types, the commitment mechanism.

3.1. *The Signal, the Norm, and the Commitment Mechanism.*

The Signal. In the basic model, a seller has only private information of his type. In the extension, besides the type, he privately receives a signal about the quality of his widget when it has been produced. The signal, denoted by $\widetilde{m}$, is either "n" ("nice") or "u" ("useless"), distributes as follows:

$$\Pr(\widetilde{m} = n|\widetilde{v} = v) = 1; \Pr(\widetilde{m} = n|\widetilde{v} = 0) = 1 - \tau \text{ and } \Pr(\widetilde{m} = u|\widetilde{v} = 0) = \tau < 1.$$

τ measures the informativeness of the signal. If $\tau = 0$, it is completely uninformative and we go back to the basic model. If $\tau = 1$, sellers would know precisely the quality of the widgets, and there would be no interesting stationary Markovian dynamics (see footnote 13). So $\tau < 1$ is assumed.

The timing is the same as that in the basic model, except that at stage 3 (see page 7), after the widgets are produced and before they are sold, sellers receive the signals.

Additional assumptions are introduced.

Assumption 3: $\frac{1-r}{(1-rq)rq} < \frac{qv-c}{qv}$.

The assumption complements Assumption 1 and says that the discount rate, r, is close enough to 1. Its significance will be clear when Proposition 3 is proved in Subsection 3.2.

Assumption 4: $p_{hf} \leq p_h$ for any h.

The assumption states that a failure always damages names. It deleted the equilibria where the meaning of failure features a U-turn, namely, for some names failure is bad ($p_{hf} < p_h$) and for some it is good ($p_{hf} > p_h$).[11] Those equilibria are unrealistic since this U-turn poses a great difficulty for buyers to coordinate their beliefs as to the meaning of failure.

The Norm of Setting Honest Prices. Consider how to utilize the post-production private information. It is not transmitted by the names, which were bought before the signals arrive. If this information is to be utilized at all, it must be transmitted through the prices of the widgets. In particular, sellers must be incentivized to set price 0 for those widgets that they know are useless, even though they would prefer a higher price. Besides the price of the widgets, sellers care only for the resale value of their names. The incentive, therefore, must consist in the gain in the resale value made by setting price 0, or equivalently, the loss in the value made by setting a dishonest high price.

The loss is imposed by the following off-equilibrium belief. Suppose buyers believe that a name that ever set a positive price for a useless widget keeps producing, and overpricing,

[11] There are no equilibria where $p_{hf} > p_h$ for any h, which would invite too much entry of bad types and hence violate the no Ponzi condition.

useless widgets. Then, no seller would want this name (so the belief is off-equilibrium); the name's resale value would thus be destroyed. This belief essentially subject names to a *norm of setting honest prices*: whenever a name sets a positive price for the useless widget, it breaks the norm and is discarded. The norm (namely the belief) punishes the sellers who set an off-equilibrium (high) price for knowingly useless widgets by destroying their names.

The Commitment Mechanism. The punishment, however, does not always succeed in binding the sellers of knowingly useless widgets to set the equilibrium price, 0. It fails, if the names' resale values are lower than the gain from setting a dishonest high price. Imposing the norm upon these names merely destroys them, which are resources of the economy, but helps nothing to utilize the post-production information. In the socially best equilibria, therefore, the norm is imposed upon only those names for which the resale value is larger than the highest gain from cheating. This gain equals the highest widget price that buyers will ever accept: $\overline{w} \equiv E(\widetilde{v}|G, \widetilde{m} = n) = \frac{qv}{q+(1-q)(1-\tau)}$, where G represents the condition that the widget is produced by a good seller. The present resale value of an h-name after producing a useless widget is rp_{hf}. Therefore, the norm is imposed upon only these h-names that $p_{hf} \geq \frac{\overline{w}}{r}$.

Buying these names thus creates an ex ante commitment to pricing the widgets honestly. Only good sellers are willing to make the commitment, and therefore, are sorted out by these names. This is a new mechanism of sorting, called the *commitment mechanism*; in the basic model, names work through the *value-adding mechanism*: good sellers outbid bad ones in competing for reputable names, because the former can add (or not destroy) value to the names. Accordingly, names such that $p_{hf} \geq \frac{\overline{w}}{r}$ are "*commitment names*", and names such that $p_{hf} < \frac{\overline{w}}{r}$ are "*non-commitment names*". The price of a non-commitment name's widget depends only on the name's history (by (2)), which is public information, whereas the price of a commitment name's widget additionally signals the seller's private knowledge of its quality. This complements Milgrom and Roberts (1986)'s research on price signalling quality.

The next subsection examines the link between social efficiency and the dynamics of name values and then formulates the problem of finding the socially best equilibria.

3.2. *The Efficiency and the Dynamics.* In any equilibrium, the dynamics of name values is decided by the no Ponzi condition and following two incentive compatibility constraints. (E1): Good sellers obtain the same return $R_G \geq 0$ from any names they buy on the equilibrium path; and (E2): bad sellers obtain 0 return on the equilibrium path and non-positive return off the path. (E2) is proved in Lemma 1, which only depends on Assumption 1 and the no Ponzi condition, and thus holds true in the extension.

Lemma 4 *The efficiency (R_G) is related to the dynamics as follows. For non-commitment h-names,*

$$R_G = rq(p_{hs} - p_{hf}) - \max\{p_h - \pi - rp_{hf}, 0\} \tag{3}$$

And for commitment h-names,

$$R_G = -p_h + \pi + rqp_{hs} + r(1-q)\tau p_{hf} \tag{4}$$

The proof is in Appendix C. Note that by Assumption 4, bad sellers never buy commitment names: otherwise they have to set $w_h = 0$ and get $-p_h - c + rp_{hf} \leq -(1-r)p_h - c < 0$; this is exactly the commitment mechanism.

An equilibrium dynamics of name values is a function $p : H \longrightarrow R^+ \cup 0$, where H is the set of all possible histories, such that (a): $p_\phi = 0$; (b): for non-commitment names, (3); (c): for commitment names, (4); and (d): the no Ponzi condition.

Any equilibrium dynamics $\{p_h\}_{h \in H}$, from which the value of other equilibrium variables, $\{w_h, \lambda_h; e_j\}_{h \in H, j=G,B}$ can be derived, uniquely decides an equilibrium.[12] Hereinafter, we only consider the dynamics without fully spelling the equilibria, unless necessary. The problem of finding the socially best equilibria is to construct an equilibrium dynamics that bears the largest R_G. That is,

Problem 1 $\max_{\{p_h\}_{h \in H}} R_G$, *s.t. (a), (b), (c), and (d).*

Notice that any equilibrium of the basic model that involves only non-commitment names is also an equilibrium in the extension, since non-commitment names satisfy the same constraints in both models. In particular, TSE-1, which implements $R_G = rq\pi$, is

[12]Given $\{p_h\}_{h \in H}$, for non-commitment names, $w_h = \min\{p_h - rp_{hf}, \pi\} + c$ and $\lambda_h = \frac{w_h}{qv}$. For commitment names, $w_h = 0$ or $\overline{w}$, depending on the signal; $\lambda_h = 1$. See the proof of Lemma 4. $e_B = \frac{1}{1-\gamma} \sum_h \rho_h (1 - \lambda_h)$, and $e_G = \frac{1}{\gamma} \sum_h \rho_h \lambda_h$, where ρ_h, the steady state mass of h-names, is decided by the dynamics.

an equilibrium in the extension. Therefore, the maximum is no smaller than $rq\pi$, which Proposition 2 states is the highest efficiency implemented in the basic model. We are looking for equilibria that implement $R_G > rq\pi$, namely, those that strictly improve over the basic model. These equilibria are called "*Norm Equilibria*", since it is the norm of setting honest prices that makes them possible.

3.3. *The Existence and Properties of Norm Equilibria.* As in the basic model, the no Ponzi condition implies that $p_h < \frac{\pi}{1-r}$ by Lemma 3, the proof of which depends only on the two equilibrium conditions (E1) and (E2), not on any specific dynamic equations, and thus holds true in the extension. Therefore, $P = \sup\{p_h | h \in H\}$ is well defined for any given equilibrium. In Norm Equilibria, $P \geq \frac{\overline{w}}{r}$; otherwise, there are no commitment names and the norm has no bite. Moreover, the value of non-commitment names strictly increases with success.[13] That is,

$$p_{hs} > p_h \tag{5}$$

We are going to prove that Norm Equilibria exist if and only if the post production signal is informative enough. For that purpose, we establish an inequality that relates P to R_G.

Lemma 5 *In a Norm Equilibria,* $P \leq \frac{\pi - R_G}{1 - rq - r(1-q)\tau}$.

Proof. See Appendix C. ■

For an intuition, consider the case where $P = p_h$ for some h-names and $r = 1$. These names are commitment names; otherwise, $p_{hs} > P$ by (5), a contradiction. Good sellers buying the names obtain the full surplus they create, π, since the names sort out good types through commitment mechanism. On the other hand, since the names are of the top value, they obtain no capital gain in any case; but they lose the names' value with probability $(1-q)(1-\tau)$, when they receive signal "n" for the useless widgets and unintentionally overprice them. Therefore, the return $R_G \leq \pi - (1-q)(1-\tau)P$. The intuition also helps us find the socially best equilibria.

[13] By (3), $p_{hs} = p_{hf} + \frac{R_G}{rq} + \frac{\max\{p_h - \pi - rp_{hf}, 0\}}{rq}$. In Norm Equilibria, $\frac{R_G}{rq} > \pi$ by definition. Thus, $p_{hs} \geq \frac{R_G}{rq} > p_h$ if $p_h \leq \pi$. For $p_h > \pi$, notice that $p_{hf} + \frac{\max\{p_h - \pi - rp_{hf}, 0\}}{rq}$ is minimized at $p_{hf} = \frac{p_h - \pi}{r}$. Thus $p_{hs} > p_h - \pi + \frac{R_G}{rq} > p_h$.

The lemma paves the way to show the necessary and sufficient condition for Norm Equilibria to exist.

Proposition 3 *Norm Equilibria exist if and only if* $\tau > \tau_c \equiv \frac{(qv-r\pi)(1-rq)}{r(1-q)[qv-(1-rq)\pi]}$.

Proof. The necessary part is proved here. By definition, $rq\pi < R_G$ in Norm Equilibria. Then $P < \frac{\pi - rq\pi}{1-rq-r(1-q)\tau}$ by Lemma 5. On the other hand, we saw $P \geq \frac{\overline{w}}{r}$. Therefore, Norm Equilibria exist only if $\frac{\overline{w}}{r} < \frac{\pi - rq\pi}{1-rq-r(1-q)\tau} \Leftrightarrow \tau > \tau_c$. The proof of the sufficiency is relegated to Appendix B. ■

The proposition shows that the norm of setting honest prices would make no difference in the basic model even if it were introduced there. Intuitively, the norm is introduced only to utilize the post-production information. Imposing the norm, however, incurs costs. With probability $(1-q)(1-\tau)$, the commitment names are destroyed, incurring a social cost, when the owners unintentionally overprice the useless widgets. This cost is proportional to $1-\tau$, while the amount of the post-production information is measured by τ. Therefore, imposing the norm brings about a net gain only if τ is beyond a threshold, τ_c. $\tau_c < 1$ by Assumption 3.

By Lemma 5, $R_G < \pi$. Therefore, the first best, where $R_G = \pi$, is not implementable in the extension either. Subsequently, we examine only the case where $\tau > \tau_c$, and hence Norm Equilibria exist by Proposition 3. First, we find that they are always more complex than the TSE of the basic model

Strictly, the complexity of a dynamics is measured by its length, denoted by l, which is defined as follows. If the minimum upper bound P is never reached by any h-names, which means name values never stop growing, then define $l = \infty$. If some h-names take the top value P, then the length of the dynamics is defined as the smallest number of periods over which new names can reach the top position.[14] That is,

Definition 2 *The length of a dynamic, denoted by* l, *is* $\min\{n | \, p_h = P$ *for some* $h \in H^n\}$, *if the set is not empty; otherwise* $l = \infty$. *Moreover, if* $l < \infty$, $h \in H^l$ *such that* $p_h = P$ *are called the first top names.*

[14] Norm Equilibria involve new names. As shown in the proof of Lemma 5, the top range names are commitment names and are bought by good sellers only. They are destroyed into new names with probability β.

The length of the TSE of the basic model is 1. The improvement in efficiency comes with increment in complexity, as follows.

Lemma 6 *In Norm Equilibrium, $l \geq 2$.*

Proof. It suffices to prove that $p_h < \frac{\overline{w}}{r}$ for any $h \in H^0 \cup H^1 = \{\phi, s, f\}$, since $\frac{\overline{w}}{r} \leq P$. $p_\phi = 0$, and by Assumption 4, $p_f = 0$. Apply (3) to ϕ and rearrange, $p_s = p_f + \frac{R_G}{rq} = \frac{R_G}{rq}$. By Lemma 5, $R_G \leq \pi - (1-rq-r(1-q)\tau)P < \pi - (1-q-(1-q)\tau)P \leq \pi - (1-q)(1-\tau)\overline{w} = \pi + q\overline{w} - (q + (1-q)(1-\tau))\overline{w} = \pi + q\overline{w} - qv = q\overline{w} - c < q\overline{w}$, where the second equality applies $(q + (1-q)(1-\tau))\overline{w} = qv$. Therefore, $\frac{R_G}{q} < \overline{w}$ and $p_s = \frac{R_G}{rq} < \frac{\overline{w}}{r}$. ■

The lemma shows that the commitment mechanism alone is incapable of implementing Norm Equilibria: before names bear the commitment effect, their values have to be pushed up by enough successes through the value-adding mechanism. On the other hand, the value-adding mechanism alone cannot implement Norm Equilibria either, as the basic model shows. Therefore, the two mechanisms are *complementary* to each other.

The next subsection is devoted to study the second best dynamics, in particular their complexity. It derives the comparative static result that relates the length of the second best dynamics to the surplus generated by good types (π).

3.4. *How Long the Second Best Dynamics Have to Be.* In this subsection, to ease notations, we let $\beta \equiv (1-q)(1-\tau)$, which is the probability of a good seller producing a useless widget and receiving a good signal, so he unintentionally overprices the widget. And let $r = 1$.[15]

As $\tau > \tau_c$, the second best equilibria are Norm Equilibria and bear $P \geq \overline{w}$. The clue on how to find the second best dynamics is hinted in the intuition of Lemma 5. Suppose h-names take the top value P. We saw that these names are commitment names and sort out good sellers through the commitment mechanism. Apply (4) to these names, $R_G = -P + \pi + qp_{hs} + (1-q)\tau p_{hf}$. To maximize R_G, we want to maximize p_{hs} and p_{hf}. Therefore, $p_{hs} = p_{hf} = P$. Note that for non-commitment names, which are ruled by the value-adding mechanism, we want to p_{hf} to be sufficiently less than p_h in order to deter bad sellers, but for commitment names, which deter bad sellers through the commitment

[15] More precisely, it is the case of r close to but still less than 1. As everything below is continuous in r, we only consider what happens at $r = 1$.

mechanism, we nevertheless prefer high p_{hf} to prevent the names from being damaged by failures. Thus,

$$R_G = -\beta P + \pi \tag{6}$$

That is, in the second best, the top names' value stops varying with performance, so long as they follow the norm of setting honest prices; they are destroyed into new names only by unintended dishonesty, which occurs with probability β.[16]

Since R_G is inversely related to P by (6), the problem of finding the second best dynamics becomes:

Problem 2 $\min_{\{p_h\}_{h \in H}} P$, *s.t. (a), (b), (c), and* $P \geq \overline{w}$.

The constraint that $P \geq \overline{w}$ ensures that there are commitment names. To minimize P, on the other hand, requires the values of commitment names to be as small as possible. It follows that only the top names are commitment names in the second best equilibria; otherwise the top level commitment names could be cut off, by which there are still commitment names and meanwhile P goes down.

To find the minimum length of the second best dynamics, consider when some dynamics of $l = N$ could be a solution of Problem 2. Since $p_{hs} > p_h \geq p_{hf}$ for any non-commitment names, the first top names are s^N-names if $l = N$. Let $P(\pi, N)$ be the maximum value of p_{s^N} among all the equilibrium dynamics of $l = N$. If $P(\pi, N) < \overline{w}$, then no dynamics of length N could satisfy the constraint $P \geq \overline{w}$ and become a solution of Problem 2, and hence the second best dynamics have to be no shorter than $N+1$. $P(\pi, N) = \overline{w}$ defines an implicit function $N(\pi)$. On the two functions, we have the following lemma:

Lemma 7 $\frac{\partial P}{\partial N} > 0$ *and* $N'(\pi) < 0$.

Proof. See Appendix C. ■

By the lemma, for any given π, $P(\pi, N) \geq \overline{w}$ if and only if $N \geq N(\pi)$. That is, $N(\pi)$ is the minimum length of the dynamics that can be solutions of Problem 2 at the given π. Moreover, the lemma says that the minimum length decreases with π. In combination, the lemma leads to the following Proposition.

[16]It follows that if $\tau = 1$ and hence $\beta = 0$, the state of the top names will become an absorber of the dynamics. Then, there are no stationary Markovian Equilibria. That is why $\tau < 1$ is needed.

Proposition 4 *The smaller the surplus generated by good types (π), the longer the second best dynamics, and hence the greater the number of successes new names have to accumulate to accomplish the top reputation.*

As was said, it needs enough successes to push up new names to possess the top reputation and become commitment names. The increment accrued by a single success is, however, in the order of π: the increment generates the capital gain and contributes to the overall return of good sellers, which, by the no Ponzi condition, is in the order of the surplus they generate which decreases with the production costs. On the other hand, the threshold of name values cross which names accomplish the top reputation is fixed at $p = \overline{w}$, the biggest one-off profit by setting dishonest prices, which does not depend on the production costs since the costs have been sunk. Therefore, the smaller the surplus, the more the successes new names have to accumulate to cross the threshold, and the longer the dynamics.

Empirically, the surplus could be proxied by the profit margin and good types proxied by high-end products of an industry. Then, the proposition lays down an inverse relationship between the average profit margin of high-end products of an industry and the time span for new brand-names to be fully established in this industry. If we believe that this profit margin in the software industry is higher than that in the wine industry, the comparative statics result is consistent with the observation that it took a decade for Microsoft to build up its reputation, while it took a century for a wine brand to achieve some commensurate fame.

4. CONCLUSION

The paper presents an OLG model where names stand for nothing intrinsic. Names can still bear reputation, in the sense that past glories of a name stand for the quality of its current products, because market competition signal and sort out good types with names with past glories, through two mechanisms. One is the value-adding mechanism: good types are more capable of adding value to the names with new glories of delivering high quality products. The other is the commitment mechanism. Highly reputable names are subject to the norm of pricing the products honestly. If they break the norm they lose all the reputation. On the other hand, they keep the reputation even when they fail to deliver high quality products, so long as they honestly price the products low. Buying these names is thus an ex ante commitment of pricing the products honestly, which only good types are willing to make. So sorting happens. It is a surprise that the norm of setting honest prices plays a role in the context of purely adverse selection of the paper.

The paper shows that the dynamics of names' reputation is linked with social efficiency. Furthermore, it derives the dynamics in the second best equilibria and finds them similar to the dynamics of personal reputation.

Lastly, the paper finds that the smaller the surplus generated by good types, the longer the second best dynamics, that is, the greater the number of successes new names have to accumulate to accomplish the top reputation. This comparative statics result predicts an inverse relationship between the average profit margin of high-end products of an industry and the time span for brand-names of the industry to be fully established.

Appendix

A. *An Example of Long Dynamics in the Basic Model.* The analysis of this example applies (3), which is expounded in subsection 3.2. The dynamics involves four states, as follows. In the equilibrium good sellers buy names of all the states.

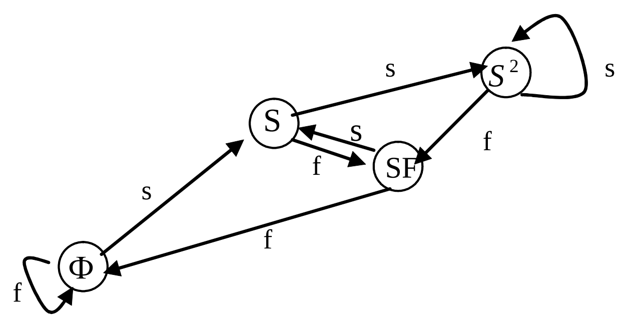

Figure 2

The Four-State Dynamics

Lemma A1. This dynamics implements the second best surplus, $rq\pi$, if and only if it degenerates to the two-state dynamics illustrated in Figure 1, with SF equivalent to Φ and S to S^2.

Proof. Consider which levels of efficiency this dynamics can implement. By (3), good sellers buying h-names obtain $R_G(h) = rq(p_{hs} - p_{hf}) - \Delta_h$, where $\Delta_h \equiv \max\{p_h - \pi -$

$rp_{hf}, 0\}$. As names of all the four states are bought by good sellers on the equilibrium path, $R_G(h) = R_G(h')$ for any h, h', that is,

$$rq(p_{hs} - p_{hf}) - \Delta_h = rq(p_{h's} - p_{h'f}) - \Delta_{h'} \tag{A1}$$

Notice that $\Delta_\phi = 0$, $p_f = 0$, $p_{s^2 f} = p_{sf}$, and $p_{s^2 s} = p_{s^2}$. Let $h = \phi$ and $h' = s$ in (A1):

$$rqp_s = rq(p_{s^2} - p_{sf}) - \Delta_s \tag{A2}$$

And let $h = s$ and $h' = s^2$, we have $rq(p_{s^2} - p_{sf}) - \max\{p_s - \pi - rp_{sf}, 0\} = rq(p_{s^2} - p_{sf}) - \max\{p_{s^2} - \pi - rp_{sf}, 0\}$. It follows that the two "max" terms are equal. They equal 0; otherwise $p_{s^2} = p_s$, which means s^2-names are equivalent to s-names: both have the same value and evolve into the same state after either a success or a failure. $\max\{p_{s^2} - \pi - rp_{sf}, 0\} = 0$ implies the following.

$$p_{s^2} - \pi - rp_{sf} \leq 0 \tag{A3}$$

Substitute $\Delta_s = 0$ into (A2), $p_{s^2} = p_s + p_{sf}$. Substitute this into (A3),

$$p_s \leq \pi - (1 - r)p_{sf} \tag{A4}$$

The surplus implemented by this dynamics is $R_G(\phi) = rqp_s$. Then, $R_G(\phi) = rq\pi \Leftrightarrow p_s = \pi$, which together with (A4) implies that $p_{sf} = 0$. Moreover, it follows that $p_{s^2} = p_s + p_{sf} = p_s$. Therefore, SF-names are equivalent to Φ-names and S^2-names to S-names. ■

B. *The Sufficiency of the Condition in Proposition 3.* To prove the sufficiency, we construct a series of Norm Equilibria whenever $\tau > \frac{(qv - r\pi)(1 - rq)}{r(1-q)[qv - (1-rq)\pi]}$. They are supported by the following series of dynamics, indexed by N. The $N-$Dynamics is illustrated below.

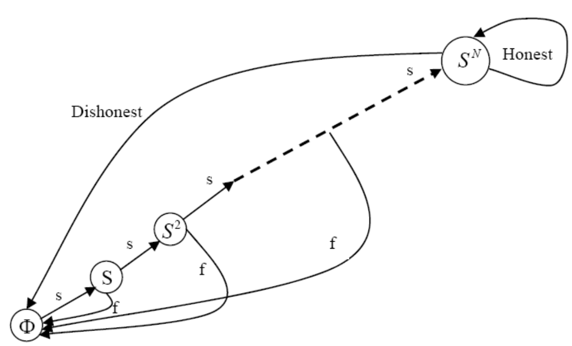

Figure 3

$N-$Dynamics

In N-dynamics, there are $N+1$ states, $S^0 \equiv \Phi, S^1, S^2 ... S^N$, all bought by good sellers. For $n = 0, 1...N-1$, S^n become S^{n+1} after a success and Φ after a failure, and are thus non-commitment names. S^N are supposed to be commitment names. The value of them does not sway with performance ($p_{s^N h} = p_{s^N}$ for any h), provided they set price 0 for useless widgets. They are destroyed into Φ-names if and only if they unintentionally price the useless widgets at $\overline{w}$.

Lemma A2. Whenever $\tau > \frac{(qv - r\pi)(1-rq)}{r(1-q)[qv-(1-rq)\pi]}$, there exists some $N_0 \geq 2$ such that for any $N \geq N_0$, N-dynamics above is in equilibrium and implements $R_G > rq\pi$.

Proof. We first figure out the p_s and p_{s^N}, according to the equilibrium constraints, and then check following two points. (1) $p_{s^N f} \geq \frac{\overline{w}}{r}$, so that S^N-names are indeed commitment names; and (2) the dynamics' R_G is strictly bigger than $rq\pi$. (2) is equivalent to $p_s > \pi$, since $R_G = rqp_s$, which is derived by applying (3) to $h = \phi$. (1) is equivalent to $p(s^N) \geq \frac{\overline{w}}{r}$, since $p(s^N f) = p(s^N)$.

$h = s^1, s^2 ... s^{N-1}$ are non-commitment names. Apply (3) to them and notice $R_G = rqp_s$ and $p_{hf} = 0$, $p(s^{n+1}) = \frac{p(s^n)}{rq} + \frac{rqp_s - \pi}{rq}$, for $n = 1, 2, ..., N-1$, where $p(s^n) > p_s > \pi$ is applied. It follows that

$$p(s^N) = \frac{p_s - \pi}{1-rq}(\frac{1}{rq})^{N-1} + \frac{\pi - rqp_s}{1-rq} \tag{A5}$$

On the other hand, as s^N are supposed to be commitment names, they satisfy (4). Substituting $p(s^N f) = p(s^N)$ and $R_G = rqp_s$ into it, we have

$$\pi - (1 - rq - r(1-q)\tau)p(s^N) = rqp_s \tag{A6}$$

From (A5) and (A6), $p_s = \frac{1-rq-r(1-q)\tau + r(1-q)\tau(rq)^{N-1}}{1-rq-r(1-q)\tau+r(1-q)\tau(rq)^N}\pi > \pi$; thus, point (2) above is checked. Moreover, when $N \to \infty$, $p_s \downarrow \pi$, and hence by (A6), $p(s^N) \uparrow \frac{1-rq}{1-rq-r(1-q)\tau}\pi$. $\frac{1-rq}{1-rq-r(1-q)\tau}\pi > \frac{\overline{w}}{r} \Leftrightarrow \tau > \frac{(qv-r\pi)(1-rq)}{r(1-q)[qv-(1-rq)\pi]}$. Therefore, whenever this condition holds, for large enough N, $p(s^N) \geq \frac{\overline{w}}{r}$ and point (1) is checked. ■

C. *The Proofs.*

In this subsection, $p(h)$ is sometime used instead of p_h to denote the price of h-names.

The Proof of Lemma 3. Given any equilibrium, let $W^t(h)$ be the sum total of the values of all the names that are generated from one unit of h-names on the t-th period after in the equilibrium. Formally, $W^t(h) = \sum_{h' \in H^t} \rho(h')p(hh')$; for example, $W^0(h) = p_h$ and $W^1(h) = q\lambda_h p(hs) + (1 - q\lambda_h)p(hf)$. These hh'-names, for $h' \in H^t$, are generated from either hs-names or hf-names on the $t-1$-th period after. Therefore, $W^t(h) = q\lambda_h W^{t-1}(hs) + (1-q\lambda_h)W^{t-1}(hf)$. The no Ponzi condition implies that $\lim_{t\to\infty} r^t W^t(h) = 0$ for any h-names.

Claim A1: $W^1(h) \geq \frac{p_h - \pi}{r}$ for any h-names.

Proof. Consider all the sellers who buy the unit of h-names. The sum of their return is $R = -p_h + \Pi_h + rW^1(h)$, where Π_h is the sum of their profits from selling the widgets. In equilibrium, $R \geq 0$ and $\Pi_h \leq \pi$. Hence, $rW^1(h) \geq p_h - \Pi_h \geq p_h - \pi$. ■

Claim A2: $W^{t+1}(h) \geq \frac{W^t(h) - \pi}{r}$ for any t and any h.

Proof. By mathematical induction. For $t = 0$, the claim is exactly claim A1. Assume the claim is true for $t = k - 1$. Then consider the case where $t = k$. Since $W^{k+1}(h) = q\lambda_h W^k(hs) + (1 - q\lambda_h)W^k(hf)$, by induction assumption, $W^{k+1}(h) \geq q\lambda_h \frac{W^{k-1}(hs) - \pi}{r} + (1 - q\lambda_h)\frac{W^{k-1}(hf) - \pi}{r} = \frac{q\lambda_h W^{k-1}(hs) + (1-q\lambda_h)W^{k-1}(hf) - \pi}{r} = \frac{W^k(h) - \pi}{r}$. Thus, the claim holds true for $t = k$. ■

The following claim is used as a technical tool.

Claim A3: Suppose sequence $\{x_t\}$ is defined as follows. $x_0 = p_h = W^0(h)$ and $x_{t+1} = \frac{x_t - \pi}{r}$ for $t \geq 0$. Then $W^t(h) \geq x_t$ for any $t \geq 0$.

Proof. By mathematical induction. $t = 0$, the claim holds true by assumption. Assume the claim holds true for $t = k$. Then consider the case where $t = k + 1$. By claim A2, $W^{k+1}(h) \geq \frac{W^k(h) - \pi}{r} \geq \frac{x_k - \pi}{r} = x_{k+1}$, where the second inequality applies the induction assumption. ■

The solution of the difference equation of Claim A3 is that $x_t = (\frac{1}{r})^t \frac{p_h(1-r) - \pi}{1-r} + b$, for some b. Then, the lemma, namely $p_h \leq \frac{\pi}{1-r}$, can be proved now. Suppose on the contrary for some h-names, $p_h = W^0(h) > \frac{\pi}{1-r}$. Then by claim A3, $r^t W^t(h) \geq r^t x_t \longrightarrow \frac{p_h(1-r) - \pi}{1-r} > 0$. That is, the no Ponzi condition is violated.

The Proof of Lemma 4. Consider non-commitment names first. No h-names are only bought by bad sellers; otherwise $w_h = 0$ and the bad sellers get $-p_h - c + rp_{hf} \leq -(1-r)p_h - c < 0$. For non-commitment h-names, $R_G = -p_h + w_h - c + r(qp_{hs} + (1-q)p_{hf}) = (-p_h + w_h - c + rp_{hf}) + rq(p_{hs} - p_{hf})$. Two subcases arise. If $-p_h + \pi + rp_{hf} > 0$, bad sellers also buy the h-names in equilibrium, and hence $-p_h + w_h - c + rp_{hf} = 0$:

otherwise, $\lambda_h = 1$, then $w_h = qv$ by (2), and bad sellers buying the names would obtain $-p_h + w_h - c + rp_{hf} = -p_h + \pi + rp_{hf} > 0$, contradictory to equilibrium condition (E2). Thus $R_G = rq(p_{hs} - p_{hf})$.

If $-p_h + \pi + rp_{hf} \leq 0$, then $\lambda_h = 1$, which implies $w_h = qv$: otherwise, $\lambda_h < 1$ and thus $w_h < qv$, and then bad sellers buying the names obtain $-p_h + w_h - c + rp_{hf} < -p_h + \pi + rp_{hf} \leq 0$. Consequently, $R_G = -(p_h - \pi - rp_{hf}) + rq(p_{hs} - p_{hf})$. The two subcases are summarized altogether into (3).

Consider then the case of commitment names. If a good seller buys a commitment h-name, with probability $(1-q)\tau$, he receives signal u, and honestly sets price 0, to keep the resale value, p_{hf}. With probability $q + (1-q)(1-\tau)$, he receives signal n and sets price $\overline{w} = E(\widetilde{v}|G, \widetilde{m} = n)$ with probability 1: suppose otherwise, concerned about keeping the resale value, he sets price 0 with probability $\mu > 0$; then conditional on price 0 and the h-names, the expected value of the widgets is $E(\widetilde{v}|h, w = 0) \propto \mu \cdot \overline{w} > 0$, and is beyond the price (0), which gives the consumers positive surplus and thus cannot happen in equilibrium. However, with probability $\Pr(\widetilde{v} = 0|G, n) = \frac{(1-q)(1-\tau)}{q+(1-q)(1-\tau)}$ his widget is actually useless and hence price $\overline{w}$ is regarded as dishonesty (though unintentionally committed), which leads the name to be destroyed; with probability $\frac{q}{q+(1-q)(1-\tau)}$, the widget is indeed useful and the name is resold at price p_{hs}. Therefore, $R_G = -p_h - c + (1-q)\tau[0 + rp_{hf}] + [q + (1-q)(1-\tau)][\overline{w} + \frac{q}{q+(1-q)(1-\tau)} rp_{hs}]$, which is simplified as (4).

The Proof of Lemma 5. The proof is similar to the proof of Proposition 2. $P \geq \frac{\overline{w}}{r} > qv$. Thus $P - c > qv - c = \pi$. In Norm Equilibria $R_G > rq\pi$. For any ε such that $0 < \varepsilon < \min\{c, \frac{R_G}{rq} - \pi\}$, find h^*-names such that $p(h^*) > P - \varepsilon$. Then, firstly, $p(h^*) > \pi$ as $P - \varepsilon > P - c > \pi$. Secondly, h^*-names are commitment names. Otherwise, by (3), $p(h^*s) = p(h^*f) + \frac{R_G}{rq} + \frac{1}{rq}\Delta_h$, which takes the minimum value when $p(h^*f) = \frac{p_h - \pi}{r}$. Therefore $p(h^*s) \geq \frac{p(h^*)-\pi}{r} + \frac{R_G}{rq} > p(h^*) - \pi + \frac{R_G}{rq} > p(h^*) + \varepsilon > P$, a contradiction again.

Then, the h^*-names satisfy (4). $R_G = -p(h^*) + \pi + rqp(h^*s) + r(1-q)\tau p(h^*f) < -P + \varepsilon + \pi + [rq + r(1-q)\tau]P \Rightarrow P < \frac{\pi - R_G + \varepsilon}{1 - rq - r(1-q)\tau}$. Let $\varepsilon \to 0$, $P \leq \frac{\pi - R_G}{1 - rq - r(1-q)\tau}$.

The Proof of Lemma 7. To ease notations, let p_n denote p_{s^n} and f_x denote $\frac{\partial f}{\partial x}$. First let us simplify the constraints (b) and (c). Substituting $p_f = 0$ into (3) for $h = \phi$, $R_G = qp_1$. Substitute this into (3) for non-commitment h-names and rearrange, we have:

$$p_{hs} = p_{hf} + p_1 + \frac{\max\{p_h - \pi - p_{hf}, 0\}}{q} \tag{A7}$$

And substitute $R_G = qp_1$ into (6) for the top names (which are the only commitment names), we have:

(A8) $$qp_1 = -\beta P + \pi$$

p_N is maximized if and only if $p_{n+1} - p_n$ is maximized for each $n = 1, 2...N-1$. For these n, s^n-names are non-commitment names as they are not in H^N and $l = N$. Apply (A7) to these names, $p_{n+1} = p_{s^n f} + p_s + \frac{\max\{p_n - \pi - p_{s^n f}, 0\}}{q}$. Given p_n, p_{n+1} is maximized by making $p_{s^n f}$ equal 0 or p_n (by Assumption 4, $p_{s^n f} \leq p_n$). Therefore, to maximize p_N, we have:

$$p_{n+1} = \max(p_n, \frac{p_n - \pi}{q}) + p_1, \text{ for } n = 1, 2...N-1$$

These $N-1$ equations define p_N as a function of p_1, π, and N. That is, $p_N = f(p_1, \pi, N)$. Obviously, $f_{p_1} > 0, f_\pi < 0$, and $f_N > 0$. Let the inverse function be $p_1 = g(p_N, \pi, N)$. That is, $P \equiv f(g(P, \pi, N), \pi, N)$, where we substitute $P = p_N$. Then $g_P = \frac{1}{f_{p_1}} > 0, g_\pi = \frac{-f_\pi}{f_{p_1}} > 0$, and $g_N = \frac{-f_N}{f_{p_1}} < 0$. Moreover, substitute $p_1 = g(p_N, \pi, N)$ into (A8), we have:

(A9) $$-\beta P + \pi = qg(P, \pi, N)$$

$P(\pi, N)$ is implicitly defined by (A9). Let $F(P, \pi, N) \equiv \beta P + qg(P, \pi, N) - \pi$. Then, $P_N = \frac{-F_N}{F_P} = \frac{-qg_N}{\beta + qg_P} > 0$, as $g_N < 0$ and $g_P > 0$, which proves the first half of the lemma.

For the second half, remember that $N(\pi)$ is implicitly defined by $P(\pi, N) = \overline{w}$. Thus, $N'(\pi) = \frac{-P_\pi}{P_N}$. We saw $P_N > 0$. To prove $N'(\pi) < 0$, it suffices to prove $P_\pi > 0$. By implicit function theorem, $P_\pi = \frac{-F_\pi}{F_P} = \frac{1 - qg_\pi}{\beta + qg_P}$. The dominator was knew to be positive. The nominator is also positive, by the claim below.

Claim A4: $g_\pi < \frac{1}{q}$.

Proof. Since $g_\pi = \frac{-f_\pi}{f_{p_1}}$ and $f_{p_1} > 0$, it suffices to prove that $-f_\pi < \frac{1}{q} f_{p_1}$. To do that, we apply mathematical induction as to N. For $N = 2$, $f(p_1, \pi, N) = \max(p_1, \frac{p_1 - \pi}{q}) + p_1 = \left\{ \begin{array}{c} 2p_1 \\ \frac{p_1 - \pi}{q} + p_1 \end{array}, \text{ if } \begin{array}{c} p_1 \leq \frac{\pi}{1-q} \\ p_1 > \frac{\pi}{1-q} \end{array} \right\}$. So $f_\pi = \left\{ \begin{array}{c} 0 \\ \frac{-1}{q} \end{array}, \text{ if } \begin{array}{c} p_1 \leq \frac{\pi}{1-q} \\ p_1 > \frac{\pi}{1-q} \end{array} \right\}$ and $f_{p_1} = \left\{ \begin{array}{c} 2 \\ \frac{1}{q} + 1 \end{array}, \text{ if } \begin{array}{c} p_1 \leq \frac{\pi}{1-q} \\ p_1 > \frac{\pi}{1-q} \end{array} \right\}$. It is obvious that $-f_\pi < f_{p_1} < \frac{1}{q} f_{p_1}$. Assume it holds true for $N = k-1$. Consider the case of $N = k$. To ease notations, we keep N but suppress other arguments p_1 and π; for example, $f_\pi(N) \equiv f_\pi(p_1, \pi, N)$. Then, $f(k) = \max(f(k-1), \frac{f(k-1) - \pi}{q}) + p_1 = \left\{ \begin{array}{c} f(k-1) + p_1 \\ \frac{f(k-1) - \pi}{q} + p_1 \end{array}, \text{ if } \begin{array}{c} f(k-1) \leq \frac{\pi}{1-q} \\ f(k-1) > \frac{\pi}{1-q} \end{array} \right\}$. So, $f_\pi(k) = \left\{ \begin{array}{c} f_\pi(k-1) \\ \frac{f_\pi(k-1) - 1}{q} \end{array}, \text{ if } \begin{array}{c} f(k-1) \leq \frac{\pi}{1-q} \\ f(k-1) > \frac{\pi}{1-q} \end{array} \right\}$

and $f_{p_1}(k) = \left\{ \begin{array}{ccc} f_{p_1}(k-1)+1 & , \text{ if} & f(k-1) \leq \frac{\pi}{1-q} \\ \frac{f_{p_1}(k-1)}{q}+1 & & f(k-1) > \frac{\pi}{1-q} \end{array} \right\}$. When $f(k-1) \leq \frac{\pi}{1-q}$, $-f_\pi(k) = -f_\pi(k-1) < \frac{1}{q} f_{p_1}(k-1) < \frac{1}{q}(f_{p_1}(k-1)+1) = \frac{1}{q} f_{p_1}(k)$, where the first inequality applies the induction assumption. When $f(k-1) > \frac{\pi}{1-q}$, $-f_\pi(k) = \frac{1}{q}(-f_\pi(k-1)+1) < \frac{1}{q}(\frac{f_{p_1}(k-1)}{q}+1) = \frac{1}{q} f_{p_1}(k)$. Therefore, $-f_\pi < \frac{1}{q} f_{p_1}$ holds true for $N = k$. ■

By the claim, $P_\pi = \frac{1-qg_\pi}{\beta+qg_P} > 0$. Therefore, $N'(\pi) = \frac{-P_\pi}{P_N} < 0$.

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