ABCs of the Propositional Calculus

0. Business Matters

Next Week: Reading Week (no class sessions or tutorials).
Next Assignment: Please read Cohen & Green’s 2004 paper, “For the Law, Neuroscience Changes Everything and Nothing.” A link to the paper is provided on the module website, which can be found here:

http://privatewww.essex.ac.uk/~wmartin/WorkObscenelyHard.html

At page 1784, Cohen and Green write:

Free will as we ordinarily understand it is an illusion generated by our cognitive architecture. Retributivist notions of criminal responsibility ultimately depend on this illusion, and, if we are lucky, they will give way to consequentialist ones, thus radically transforming our approach to criminal justice.

Write a short essay explaining how Cohen and Greene think advances in neuroscience will affect our view of free will and criminal justice. Assess whether or not their argument is convincing.

Your assignment should be submitted by email attachment to your writing tutor no later than 4pm on Monday, 29 November. This is not a summative (i.e., formally assessed) assignment, so there is no need to submit your essay to the Philosophy Department Office.

1. Overview of our Logical Agenda

The main aim of the course is to provide basic training in two formal logical calculi: the propositional calculus and predicate logic. These are formal languages that are designed to exhibit the structure of arguments and evidence, test for validity, and provide tools of inference. The plan of attack is as follows:

THIS TERM:
(1) Introduction to Basic Concepts of Logic.
(2) Recognizing Fallacies
(3) Symbolization in the Propositional Calculus.
(5) Assessing Validity in the Propositional Calculus: Natural Deduction

NEXT TERM:
(6) Symbolization in the Predicate Calculus.
(7) Assessing Validity in the Predicate Calculus.

2. Ten Basic Logical Terms

- argument
- premise
- conclusion
- proposition
- calculus
- soundness
- validity
- deduction
- induction
- logical form

3. Some Sample Arguments

i. Dave and Nick are both coalition men, so Dave is coalition man.
ii. I will probably get an A, since I almost always get an A.
iii. God exists because the Bible tells us so, and we know that what the Bible tells us must be true because it is the revealed word of God.
iv. There is no proof that the secretary leaked the news to the papers, so it must have been someone else.
v. Arsenal are the best team since they are going to win the league and if they win the league then they must be the best team.
vi. If ignorance is bliss then knowledge is terrible. Knowledge is terrible, so ignorance must be bliss.
vii. Homer, Marge, Bart and Lisa all have four fingers on each hand, so the Simpsons must be a four-fingered family.
viii. It cannot be true that God can do everything. For either he can create a stone that is too heavy for him to lift, or he cannot. If he can, then there is something he cannot do: lift the stone. If he can’t, then there is something he cannot do: create the stone.
ix. With regard to good and evil, these terms indicate nothing positive in things considered in themselves, nor are they anything else than modes of thought, or notions which we form from
the comparison of one thing with another. For one and the same thing may at the same time be both good and evil or indifferent. Music, for example, is good to a melancholy person, bad to one mourning, while to a deaf man it is neither good nor bad. (Spinoza, Ethics)  
x. Even though there may be a deceiver of some sort, very powerful and very tricky, who bends all his efforts to keep me perpetually deceived, there can be no slightest doubt that I exist, since he deceives me; and let him deceive me as much as he will, he can never make me nothing as long as I think that I am something. Thus, after having thought well on this matter, and after examining all things with care, I must finally conclude and maintain that this proposition: *I am, I exist*, is necessarily true every time that I pronounce it or conceive it in my mind. (Descartes, Meditations)

4. Validity vrs Soundness

We distinguished between two forms of praise for arguments.

VALIDITY: To say that an argument is valid is to make a conditional claim. If the premises of a valid argument are true, then the conclusion must be true as well.

SOUNDNESS: To say that an argument is sound is to say both that it is valid and that it has true premises.

Notes:

a. Soundness is the highest virtue of deductive argument.
b. Soundness is validity plus true premises.
c. All sound arguments are valid.
d. But not all valid arguments are sound.
e. Why? Because a valid argument might have false premises.
f. An argument can have true premises and a true conclusion, but not be valid or sound.

-- Examples

All men are mortal.
Socrates is a man.
∴ Socrates is mortal.

This argument is valid and sound.

All men are tornadoes.
Socrates is a man.
∴ Socrates is a tornado.

This argument is valid but not sound.

All men are mortal.
Socrates is mortal.
∴ Socrates is a man.

This argument has a true premises and reaches a true conclusion, but it is neither valid nor sound.

What about this argument? Is it valid or invalid? sound or unsound?
All Manchester United Players are professional footballers.
All professional footballers are millionaires.
∴ All Manchester United Players are millionaires.

5. Logical Form

One of the most central concepts in formal logic is the notion of logical form. Its proper definition, however, has proved to be both elusive and contentious. We shall return in due course to consider how best to define “logical form”, but it is not difficult to gain a preliminary understanding of this notion. Consider three arguments

(A) If it is Thursday today then it is Friday tomorrow. It is Thursday today. Therefore it is Friday tomorrow.
(B) If it is Saturday today then it is Sunday tomorrow. It is Saturday today. Therefore it is Sunday tomorrow.
(C) If Dave is a Martian then Nick is too. Dave is a Martian. Therefore Nick is too.

These arguments share no common premises or conclusions, yet clearly they have something in common. In particular, once we abstract from the particular subject matter of the arguments (days of the week, politicians) and from the specific claims made concerning that subject matter (that today is Sunday, that Blair is a Martian), we see that all three arguments exhibit a common underlying structure:

If \( p \) then \( q \). \( p \). Therefore \( q \).

We call this underlying structure the form of an argument (as opposed to its content or matter). This particular logical form is so fundamental to proof that it has its own name: *modus ponens*.

Logical form is the primary concern of formal logic. In particular, the logician characteristically abstracts from the particular subject matter or content of an argument in order to assess its form. In particular, logic’s central task is to identify those logical forms that are valid or truth-preserving. One of the first skills to learn in logic is the skill of recognizing the logical form of arguments expressed in natural language prose.
6. Deduction vrs. Induction
We distinguish between two basic genera of argument. Most of the arguments we will be concerned with in this course are deductive arguments. A successful deductive argument guarantees the truth of its conclusion, given the truth of its premises. In other words, a successful deductive argument must be valid. Here are examples of deductive arguments:

If all men are mortal and Socrates is a man then Socrates is mortal.
If Plato and Aristotle are philosophers then Plato is a philosopher.
If Descartes is a politician and all politicians are honest then Descartes is honest.

Inductive arguments provide evidence for their conclusions without providing this sort of guarantee, usually through the accumulation of instances. If you examine ten thousand rubies and find that all of them are red then that provides inductive support for the conclusions that all rubies are red, and that some particular as-yet-unobserved ruby is red. The inductive evidence supports these conclusions, but it does not guarantee their truth. Nothing in the premises of this reasoning about rubies makes it impossible that the next ruby will be a rare white one.

7. Formulae of the Propositional Calculus
Over the next few weeks we will be learning to construct and assess proofs in an artificial symbolic language known as the Propositional or Sentential Calculus. (These two terms are synonymous.) It is called the propositional calculus because it takes as its basic units propositions or sentences. These propositions express some state of affairs; they are descriptive declarative statements that are either true or false. That is, they are truth-evaluable. Since questions and imperatives and sentence-fragments are not truth-evaluable, they do not figure in the propositional calculus. A sentence in the language of the Propositional Calculus is called a formula.

8. Sentential Constants, Sentential Variables, Logical Symbolism
Already in the foregoing discussion of logical form we have begun a practice that will be central to the skills developed in this course: symbolization. Just as algebra uses letters like x and y to represent numerical values, and f(x) to represent functions, so in logic we will use letters and other symbols to represent logical items.

We shall use upper case letters (P, Q, R, …) as sentential constants -- as symbols for particular propositions.
We shall use the TILDA (~) to mean “not” [NEGATION]
We shall use the AMPERSAND (&) to mean “and” [CONJUNCTION]
We shall use the WEDGE (v) to mean “or” [DISJUNCTION]
We shall use the ARROW (→) to mean “if … then …” [CONDITIONAL]
We shall use the DOUBLE-ARROW (↔) to mean “if and only if” [BICONDITIONAL]

Formula of the predicate calculus can either be atomic sentences (represented by a single sentential symbol) or they can be compound – constructed by combining sentential symbols with one or more of the five connectives. Examples. Let P and Q represent the atomic sentences “Brown is a politician” and “Cameron is a politician.” In that case:

~P represents the compound sentence “Brown is not a politician”
P & Q represents the compound sentence “Both Brown and Cameron are politicians”
P → Q represents the compound sentence “If Brown is a politician then so is Cameron”
P v Q represents the compound sentence “Either Brown is a politician or Cameron is.”
P↔Q represents the compound sentence “Brown is a politician if and only if Cameron is.”

Compound sentences can themselves be compounded. When doing so the predicate calculus uses parentheses to mark the scope of the connectives. E.g.:

(P v Q) → ~(~P & ~Q)

Connectives within parentheses have limited scope. The main connective of a sentence is the one whose scope encompasses the whole sentence.

9. Elementary Translation Exercise: With these basic rules, we can begin to translate some sentences of English into the Predicate Calculus. A translation should begin with a key, which specifies the content of atomic sentences; it then uses those atomic sentences to construct compound sentences using the connectives and parentheses.

(i) Peter is happy and Quentin is happy.
(ii) Either Peter is happy or Quentin is not happy.
(iii) If Peter is happy then either Quentin or Rebecca is happy.
(iv) Peter is happy if Quentin is happy.
(v) Rebecca is happy and both Peter and Quentin are happy.
(vi) Peter is happy if and only if Peter is happy.
(vii) Quentin and Peter are both happy if Mary is happy.
(viii) Peter is happy or it is not the case that either Peter or Rebecca is happy.
(ix) It is not true that Peter, Quentin and Rebecca are all happy.
(x) Peter is not happy if Quentin is happy.

10. **Inclusive and Exclusive Disjunction:** In English and other natural languages, the word “or” is ambiguous. Consider the sentence: “Either Paul or Pat will object to this proposal.” This might mean that one of the two will object **but not both**. (This is the exclusive “or”.) But it might mean that at least one of the two will object, **and perhaps both** will. (This is known as the inclusive “or”, since it includes or allows the possibility that both disjuncts are true. In the Propositional Calculus, the wedge symbol is standardly taken to represent inclusive disjunction. That is, a disjunctive formulae is true if either of its atomic sentences is true or if both are. It is only false if both atomic sentences are false.

12. **Here is a Slightly Harder Exercise:** Using the wedge to represent inclusive disjunction, propose a formula of the predicate calculus which represents the exclusive reading of “Either Pat or Paul will object to this proposal.”

13. **Fallacies:** A fallacy is an error of reasoning involving something other than the use of false premises. Fallacies are traditionally divided into formal and informal fallacies. An argument commits a **formal fallacy** if it relies on an invalid argument form. Hence a formal fallacy is a bit like a mistake in calculation in mathematics. **Informal fallacies** appear not so much in deductive proof but in various forms of persuasive speech (i.e., in political rhetoric, leading articles and editorials, advertising, debate) – that is, anywhere language is used in an attempt to persuade people what to do or think. The logical ideal is that such attempts at persuasion proceed by advancing valid arguments in support of the conclusion being defended. But of course many acts of persuasion fall short of this ideal. The catalog of informal fallacies is an attempt to identify what goes wrong. Over the rest of this term we will build up a catalog of such informal fallacies. We begin with three particularly famous ones.

**Straw Man:** An argument attacks a straw man when it is directed at some unduly weakened or vulnerable version of the proposition or proposal actually under dispute. *The white paper proposal to check the power of Local Education Authorities is fundamentally flawed. We must not abandon the principle that local representatives maintain local control over local schools. After all, it is local people who understand local conditions, and can be called to account for local policies.*

**False Dilemma:** An argument turns on a false dilemma when it misleadingly suggests that only two (usually extreme) possible positions are possible on a particular question. *Either we stick it out in Iraq or we effectively surrender to International Terrorism. To surrender to terrorism will make matters far worse than they are now, so we simply have to stick it out and resist the instinct – however natural it may be – to cut and run when the going gets tough.*

**Ad Hominem:** An argument is ad hominem (against the man) when it is directed against the person or institution advancing a thesis or proposal rather than against the thesis or proposal itself. *Who is opposing this legislation? The record is clear. The money to fund the opposition to the smoking ban comes from big multinational tobacco companies, who have a vested interest in preserving the status quo – regardless of its health consequences. Our democracy is not for sale at any price, and neither is the health of our citizenry. I urge the House to stand firmly with the Government in supporting this bill to keep smoke out of public places – including public houses.*